Unsecured Indebtedness in the United Kingdom – implications from a rational agent model*

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Summary

This report is the end of project submission for the *Understanding Debt* project undertaken by NIESR for the DWP and DTI. The report focuses upon undertaking a rigorous analysis of private sector indebtedness, and unsecured debt in particular. Key objectives include:

- to explore the interactions between saving and unsecured indebtedness that are derived from an articulated and carefully calibrated rational agent model of the household,
- to consider the extent to which the current accumulation of debt can be explained as the product of rational responses to the prevailing economic environment,
- to explore the determinants and prevalence of over-indebtedness when individuals make sensible financial decisions,

Key findings of the current report are:

- rational households take on unsecured debt when they expect incomes to rise in the future – in this context debt is a positive mechanism allowing income smoothing over the lifecycle.
- the rational agent model substantially under-states both non-pension wealth and unsecured debt reported for the private sector in the National Accounts. Although it is conjectured that the disparity for non-pension wealth could be resolved by including a bequest motive, best estimates for the effects of adjustments to reconcile the levels of unsecured debt suggest that the model would still understate the National Accounts aggregate by 30%.
- unsecured debt generated by the rational agent model tends to arise as the product of low incomes, which in turn are due to a marital dissolution or involuntary unemployment. These observations are qualitatively consistent with survey data, and emphasise the importance of "indirect drivers" of private sector indebtedness.
- the rational agent model suggests that the trend toward a higher educated population should result in greater demand for credit early in any given birth cohort's lifetime, and larger debts for those who are indebted. If this result is valid, then it may provide a motive to expand the role of student loan programmes.
- the rational agent model implies that approximately one fifth of all households should experience at least one year of over-indebted and poverty during their lives. This result implies that *over-indebtedness and financial deprivation cannot simply be dismissed as a product of poor financial planning.*

1 Introduction

Unsecured indebtedness in the UK personal sector has risen sharply during the last half century, from 6.3% of GDP in 1950 to 17.7% of GDP – £234 billion – today (2007Q1).[1] This trend, matched by the experience of other G7 countries, has been the subject of growing public concern.[2] Nevertheless, it remains unclear whether current borrowing patterns should excite alarm. The trend toward higher household debt may, for example, be a rational response to the favourable economic conditions that have prevailed during recent decades, and the increasingly liberal terms of personal borrowing. This report explores the interactions between household savings and indebtedness that are described by a rational agent model, under the assumption that individuals are perfect financial planners. Given credible assumptions regarding the economic environment, we also consider how closely the behaviour generated by our rational agent model reflects the pattern of unsecured debt that is currently observed in the UK.

The related literature

A large literature has tested the ability of the life-cycle framework to capture observed aspects of consumer behaviour.[3] Most of this literature has, however, ignored the specific effects of consumer credit on consumption decisions by assuming either that individuals cannot borrow, or that they have infinite available credit. In the former case, consumer credit is explicitly omitted, and in the latter debt affects consumption only to the extent that it determines the lifetime resources that are available for allocation. For debt to have a prominent influence on consumer behaviour under the life-cycle model, borrowing must be rationed so that it is either subject to strict limits, or to higher interest rates than those earned on financial assets.

Since the mid 1980's, a great deal of work has been done to consider the influence of liquidity constraints on consumption and savings behaviour in the context of uncertain future labour income.[4] Simulation studies have suggested that liquidity constraints can affect the consumption behaviour of individuals, even if they have a low probability of ever actually being constrained, and theoretical advances mean that the underlying processes are now well understood.[5] Furthermore, empirical analyses

[1] Solomou & Weale (1997) report that personal sector indebtedness in the UK was £2.175bn in 1950. Personal sector unsecured indebtedness for 2006Q1 reported by the Office for National Statistics, ONS (calculated as the sum of codes NNRG, NNRK and NNRU). GDP in the UK was £13.162bn in 1950, and £1,319.485bn for the 12 months to 2007Q1, as reported by the ONS, code YBHA.

[2] See, for example, articles in the *Financial Times*, 28/07/2006 "Banks see rise in bad debts", and the *Daily Express*, 27/09/2006 "Brits 'have worst debts in Europe'". Similarly, in the Australian broadsheet, *The Age*, 12/11/2006, "Debt hell for middle class".

[3] See Browning & Crossley (2001) for a review.

[4] Carroll (2001), p.27, cites Zeldes (1984) as the first study in this field.

[5] See Zeldes (1989*b*) for an early example of simulation work, and Low (2005) for a contemporary example. Regarding the theoretical underpinnings of precautionary savings motives, see Carroll & Kimball (1996) for behavioural responses in the absence of liquidity constraints, Carroll & Kimball (2001) when households are not permitted to borrow, and Fernandez-Corugedo (2002) when differential interest rates apply to savings and borrowing (so-called "soft constraints").

suggest that liquidity constraints play an important role in determining consumer behaviour in practice (e.g. Zeldes (1989*a*) and Gross & Souleles (2002) using US data, and Stephens (2006) and Benito & Mumtaz (2006) for recent studies using UK data). Gross & Souleles (2002), for example, use a detailed panel data set compiled by several different credit card providers in the US to determine the influence of credit supply on consumption. They find that the average marginal propensity to consume out of liquidity is strongly significant, ranging between 10 and 14 percent. They also find that the MPC is higher for individuals who are close to their credit limit (consistent with a binding liquidity constraint), and lower but significant for those starting well below their credit limit (consistent with a precautionary savings motive).

Key objectives of the analysis

The primary purpose of the current study is to explore the interactions between saving and unsecured indebtedness that are derived from an articulated and carefully calibrated structural model of the household. The model is 'articulated' in the sense that a great deal of effort has been spent in obtaining a reasonably detailed reflection of the characteristics that have been cited in survey data as key drivers of unsecured indebtedness. It is a structural model in the sense that it is based upon an explicit behavioural framework in which agents – households in the current study – are considered to make their decisions regarding saving and labour supply to maximise expected lifetime utility, subject to the various uncertainties that influence their future circumstances.

It is hoped that this analysis will help to inform policy makers regarding the motives that provide potential explanations for patterns of behaviour toward unsecured indebtedness and saving more generally. Furthermore, by comparing the simulated behaviour against data regarding actual consumer credit use, the study contributes to the substantial literature that tests whether the rational agent life-cycle model provides an adequate representation of real-world behaviour, or in what ways it may be deficient. Comparisons with survey data will also help to identify whether – given plausible assumptions regarding the economic environment – contemporary consumer borrowing may be imprudent.

Analytical approach

Indebtedness bears upon the financial circumstances of individuals in three important respects. It is a liability on the lifetime resources that are available for consumption, it draws individuals closer to any period specific liquidity constraint that they may face, and it influences implicit rates of return to the extent that it is used to finance investment. This study focuses primarily on unsecured indebtedness, omitting incentives for financial leveraging. The current focus reflects the observation that repayment difficulties (and hence liquidity constraints) are strongly associated with unsecured indebtedness, with consequent implications for consumption and welfare.[6]

The stochastic dynamic programming model that is used to undertake the analysis has been carefully designed to capture key factors that influence household indebtedness. Households are subject to both hard and soft liquidity constraints, in the sense that the interest rate charged on debt is an increasing function of a household's debt to income ratio, and each household is subject to an age specific credit limit. This specification implies that simulated households may experience a downward spiral of increasing indebtedness and higher interest charges, which influences their precautionary savings motive. Households are considered to make consumption and labour supply decisions to maximise expected lifetime utility, subject to uncertainty regarding their labour income, marital status, number of dependent children, and time of death. This model is calibrated to survey data, and is used to explore the interaction between indebtedness, saving, and labour supply.

Outline of the report

Section 2 presents statistics that place current levels of household debt in their historical context, and describes the contemporary distribution of indebtedness. Section 3 describes the structural dynamic programming model that we use to analyse behaviour towards indebtedness, and Sections 4 and 5 report the model calibration. The paper has been structured so that the policy focussed reader may omit the subsections of Section 3 (a review of the introductory discussion is encouraged), and Sections 4 and 5 without handicap. A brief review of the intuition behind the life-cycle model, as it relates to household saving and indebtedness is provided in Section 6. Section 7 focuses upon describing the behaviour toward saving and indebtedness implied by our rational agent model, and relating the behaviour toward indebtedness to contemporary survey data. Implications of the model for the related issues of over-indebtedness and poverty are explored in Section 8. A summary of results and associated discussion are provided in a concluding section.

2 Recent Trends in Personal Sector Indebtedness

This section provides a brief review of statistical evidence regarding household indebtedness in the UK. We begin by describing trends in aggregate data, before reporting distributional observations for a contemporary population cross-section.

2.1 Aggregate trends

The period following the housing market crisis of the early 1990's has been particularly favourable for holding debt in the UK. Figure 1 displays five key statistics that place the current borrowing

[6] See, for example, *Inflation Report*, November 2006, Bank of England, p. 15.

environment in its historical context. The top panel of the figure reports quarterly data for income growth and fluctuations in the unemployment rate observed since 1971Q1 (the earliest date for which data are available). These data indicate that real disposable household income has grown reasonably smoothly at a rate of 2.8% per annum during the last three and a half decades, and is now 2.7 times what it was in 1971.[7] At the same time, the unemployment data reveal a substantial degree of temporal variation, peaking at values in excess of 10% during the recessions of the early 1980's and early 1990's, between troughs that are as low as 3.4% (1973Q4). In this context, the last 10 years are notable for the low and stable unemployment rates observed.

The bottom panel of Figure 1 reports 12 month moving averages for inflation, the base interest rate, and interest rate volatility. The figure makes clear that all three of these statistics have exhibited a downward trend during the past three decades, and have been relatively low and stable for the last decade. Although the highs observed following the OPEC oil crisis may be taken as something of an outlier, inflation, interest rates, and interest rate volatility were also substantially higher during the recessions of the early 1980's and 1990's than they have been for the last 10 years.[8]

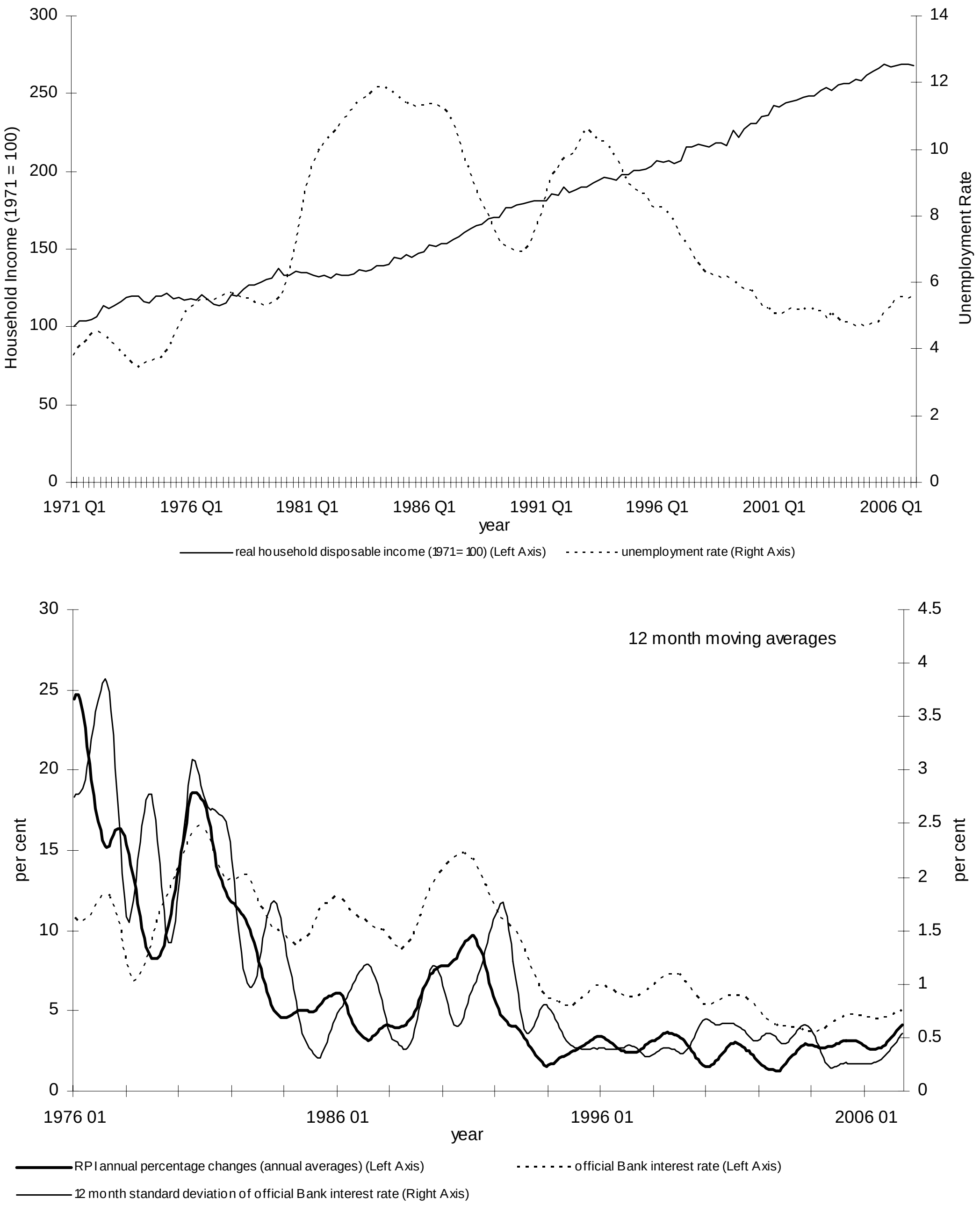
Strong income growth, low inflation, and low interest rates imply that debt is now more affordable than it was in the past. Low unemployment, and low interest rate volatility imply that debt servicing has been less risky. In this context, it is of little surprise that households have chosen to take on more debt. The extent to which household indebtedness has grown is depicted in Figure 2, which reports quarterly ratios of mortgage debt and unsecured credit to disposable income. These data reveal that mortgage debt has grown particularly strongly over the 20 year period for which data are available – the mortgage to disposable income ratio is now (2007Q1) just over twice what it was in 1987Q1. Although the ratio of unsecured debt to disposable income has exhibited more variation than that for mortgages (declining after the housing crisis of 1990), it has also exhibited strong growth, and is now more than one and a half times what it was in 1993Q4. It is of note that both debt ratios are now higher than they have been at any other time in the past 20 years.

Although the debt to income ratios reported in Figure 2 show substantial growth since 1987, the favourable economic conditions have offset the associated impact on household budgets and balance sheets. Figure 3 reports quarterly data for the mortgage debt to financial assets ratio, unsecured debt to financial assets ratio, and the aggregate debt servicing ratio.[9] The figure indicates that all three

[7] Code NRJR refers to population aggregates, and so tends to slightly overstate the growth of disposable income measured on a per capita basis.

[8] The low and stable rates of inflation that have been observed during the last five years in the UK coincide with the implementation of inflation targeting that began in 1992. See the Bank of England website (http://www.bankofengland.co.uk/monetarypolicy/history.htm) for details regarding the evolution of monetary policy in the UK.

[9] As Nickell (2004) notes, the rise in the value of household assets is predominantly due to an increase in financial assets, as the effects of the housing boom tend to cancel out from a population wide perspective.



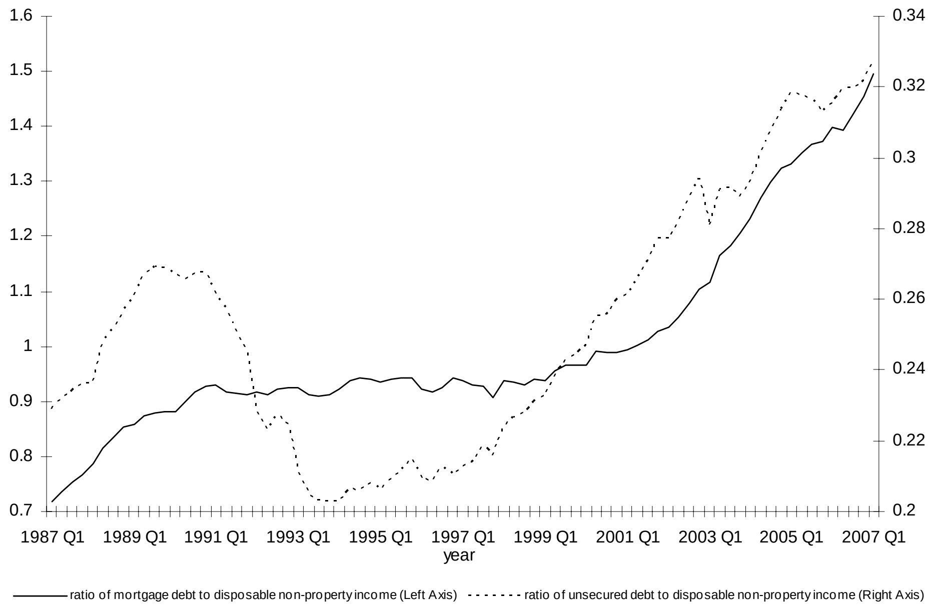
Source: Data reported by the ONS or the Bank of England

ONS codes: Real household disposable income = NRJR, unemployment rate = 100.(1 - MGRZ / MGSF), Retail Prices Index (RPI) = CZBH

BoE codes: Monthly average of official Bank of England interest rate = IUMABEDR

RPI data reported from 1976 to align to earliest date for which official Bank interest rate available

Figure 1: Historical Economic Indicators for the UK



Source: All data reported by the Office for National Statistics

ONS codes: Mortgage debt = NNRQ + NNRR + NNRS, unsecured debt = NNRG + NNRK + NNRU

Disposable non-property income = RPQK - ROYL + ROYT - NRJN + ROYH

Figure 2: Debt to Disposable Income Ratios for the UK

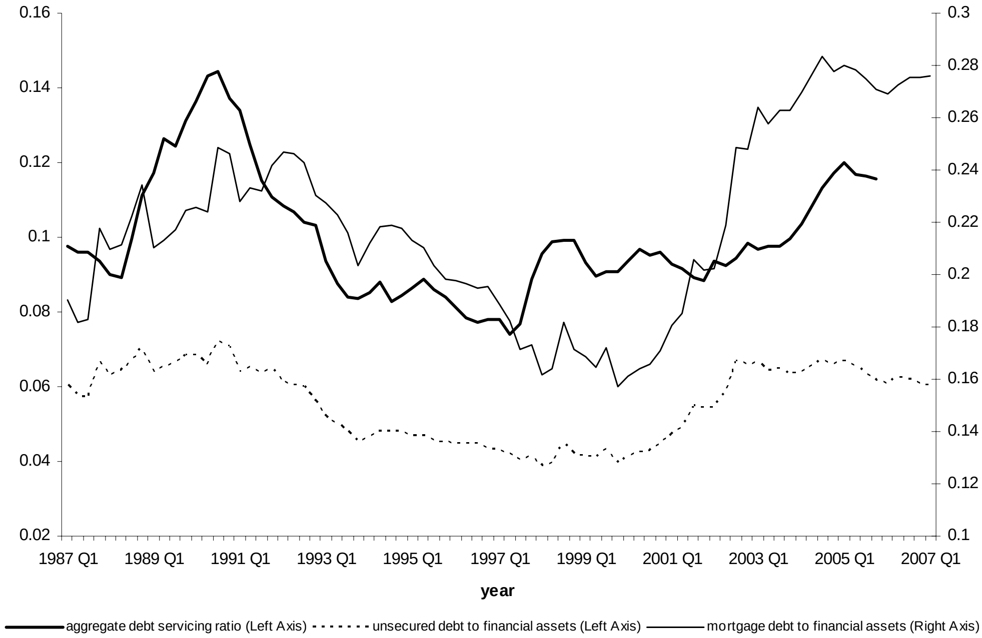
statistics fell from peaks observed in 1990Q3, with only the ratio of mortgage debt to financial assets now greater than the value it was then. Furthermore, responses to qualitative surveys suggest that people now find their debts less burdensome than in the past.[10] These statistics consequently suggest that, in spite of the well publicised rise in private sector debt, the financial well-being of households does not appear to have deteriorated substantially during the past decade.

2.2 Distributional data

Analysis of the distribution of debt amongst the UK population is complicated by the scarcity of associated micro data. At the time of writing, the richest source of micro data regarding the assets and debts held by UK households is the 2000/01 wave of the British Household Panel Survey (BHPS).[11] This survey reports the responses of just under 7000 households to a suite of questions that were designed to identify household assets and liabilities. The cross-sectional wealth data from the 2000/01 wave of the BHPS do not describe the practical experience of any population cohort. It is also well recognised

[10] See, for example, *Over-indebtedness in Britain: a DTI report on the MORI financial services survey, 2004*, see: http://www.dti.gov.uk/files/file18550.pdf.

[11] See Appendix A for further details regarding the BHPS. There are currently four main sources of wealth data for the UK; the General Household Survey (primarily for housing wealth), the Family Resources Survey, the English Longitudinal Study of Ageing (for the English population over the age of 50), and the BHPS. Following a review of these alternative data sets, the BHPS was identified as the most comprehensive source of wealth data currently available for the UK. The wealth data provided by the BHPS are due to be updated in 2007 with the release of the 15th wave of the panel. The distribution of household assets and liabilities in the UK will also be greatly enhanced when the Wealth and Assets survey is made available, early results of which are scheduled to be released in December 2007, with the main report to be released in Spring of 2009.



Source: Data reported by the ONS or the Bank of England

ONS codes: Mortgage debt = NNRQ + NNRR + NNRS, unsecured debt = NNRG + NNRK + NNRU, financial assets = NNML

BoE codes: Debt servicing ratio – unpublished calculations by the Bank of England

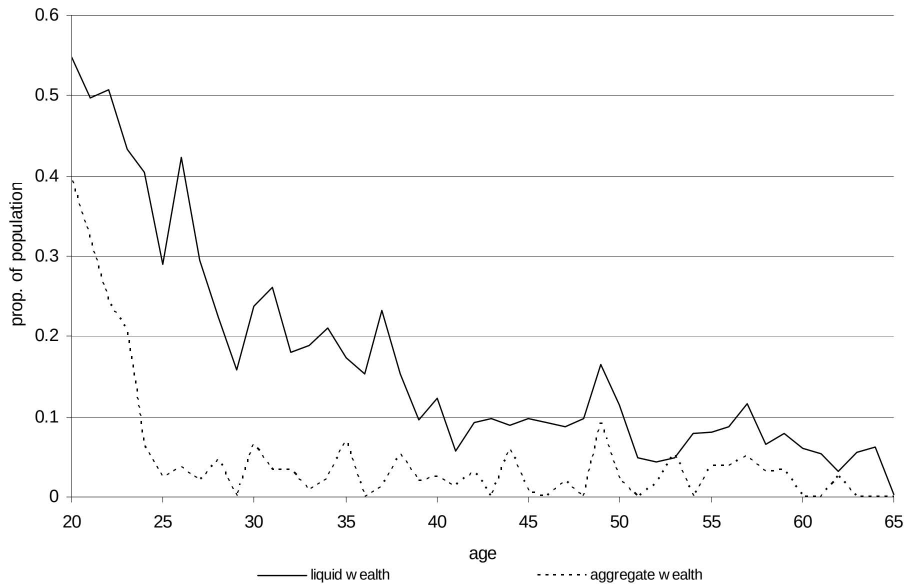
Figure 3: Debt Servicing and Debt to Assets Ratios for the UK

that the incidence of debt is commonly understated by surveys of this type, an issue that is discussed at greater length in Section 7. Nevertheless, the associated distributional statistics, which are reported here, provide a useful backdrop against which to interpret the behaviour generated by our rational agent model.

Three measures of household wealth were imputed from the BHPS data: securities wealth, housing wealth, and pension wealth.[12] Although all three measures of wealth contribute to the aggregate resources available to a household, differences in liquidity associated with each imply that they feature differently in the household budget constraint. For the simulations that are presented in Section 7, we distinguish between housing and securities wealth on the one hand, and pension wealth on the other. This distinction reflects the observation that, in practice, borrowers can use remortgaging to finance current consumption out of housing wealth, whereas it is not possible to leverage against pension wealth during the working lifetime.[13] The same distinction is consequently also used to frame the discussion that is presented here. Housing and securities wealth is hereafter referred to as *liquid wealth*, and liquid wealth together with pension wealth is referred to as *total wealth*.

[12] Wave 10 of the BHPS asked survey respondents for their personal estimates regarding the value of any residential property that they owned, the value of associated mortgages, the value of their financial investment products, and the value of any loan products. Pension wealth is highly opaque in the UK, and the BHPS did not include questions regarding the value of private pensions. Pension wealth was consequently imputed from the panel data provided by the BHPS following the approach described by Blundell et al. (2002). See the Appendix for full details.

[13] See Benito (2007) for recent UK evidence that households use housing equity to smooth over adverse economic shocks.



Source: author calculations using (weighted) BHPS survey data
liquid wealth equal to value of financial securities plus residential property
aggregate wealth equal to liquid wealth plus value of private (personal and occupational) pensions

Figure 4: Proportion of Population Net Debtors by Age – BHPS data for 2000/01

Figure 4 reports the proportion of households, by age and wealth specification, that were identified by the BHPS survey data as net debtors. This figure indicates that, if imputed pensions are taken into consideration, then very few households were identified as net debtors after age 25 in 2000/01.[14] This view, however, fails to reflect the short-term financial reality of most households. The second series displayed in the figure reveals that a substantial proportion of the population possess negative net liquid wealth, falling from a peak of more than half the population at age 20, to 10% by age 39, and then gradually falling with age so that negligible households are identified as net debtors by age 65[15]. This suggests that a substantial proportion of the working aged population make use of credit, particularly when young.

To describe the size and distribution of the debts that were reported by households in the BHPS, and how these relate to assets in the survey, Figure 5 displays the average net wealth of households, by age group and wealth quintile. The two panels of the figure relate closely to the statistics that are reported in Figure 4. Taking, for example, the lowest age band reported in Panel A of Figure 5, households in the bottom two wealth quintiles have debts that exceed their liquid assets, while households in the third quintile have debts that are approximately equivalent to their liquid assets. This is consistent with the statistics displayed in Figure 4, which indicate that 40 to 50 percent of households between ages 20 and

[14] Less than 2.5% of households between ages 25 and 65 are identified as possessing total assets of lower value than their total debts.

[15] 0.27 percent

24 are debtors, measured with regard to net liquid wealth.

The statistics reported in Panel A of Figure 5 indicate that the net debts described by the BHPS data are distributed highly unequally, with the vast majority of net debt held by the lowest population quintile. In both panels of the figure, average net assets tend to rise with age, and only the lowest population quintile holds negative liquid assets on average throughout most of the lifetime considered in the figure. Nevertheless, the average net debt (based on liquid assets) of the lowest population quintile falls by more than 40% from ages 20-24 to 25-29, and then by 40% again to ages 30-34. Hence, net debt appears to be heavily concentrated in the poorest fifth of young households.

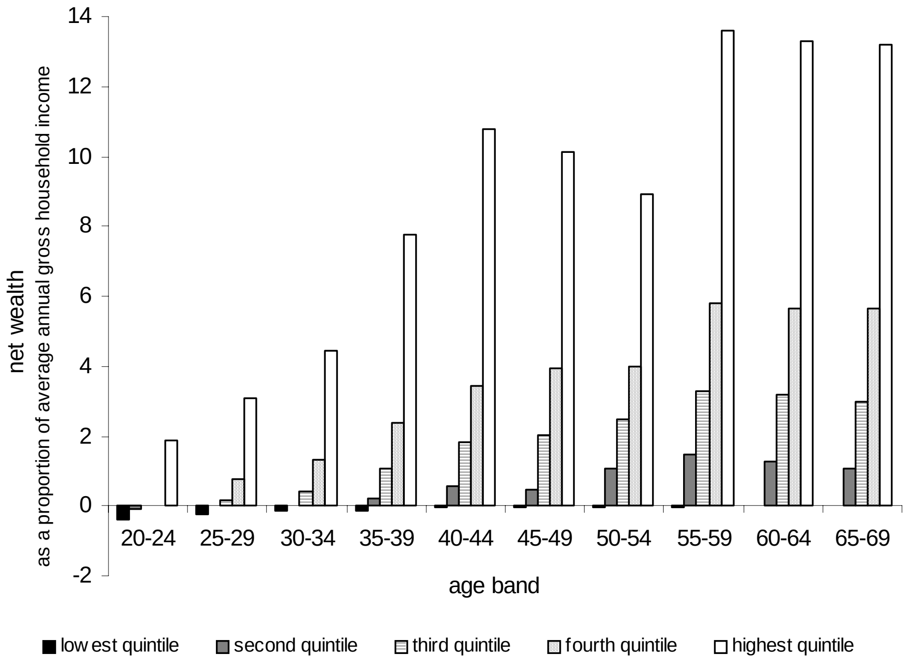
Perhaps more conspicuous in Figure 5 is the disparity with which household wealth is distributed. The average net liquid wealth of the highest population quintile is approximately twice that of the next most wealthy quintile for most of the age groups reported in the figure, for measures of both liquid and total net wealth. Hence, the distribution of net wealth is skewed to the left, with few households characterised as net debtors, the second to fourth population quintiles bunched relatively tightly, and the most wealthy 20 percent of households holding a substantially disproportionate share of net wealth.

To obtain further detail regarding the distribution of debt amongst UK households, Figure 6 distinguishes the population with regard to whether any household member was recorded as having received an 'A-level' qualification or higher, which split the full sample into two roughly equal halves.[16] The statistics reported in Figure 6 paint an interesting picture: although higher educated households are wealthier on average, the largest net debts tend to be held by young households that are higher educated. Notably, these observations are consistent with the life-cycle hypothesis, which states that more pronounced borrowing should be observed amongst households with expectations of higher income in the future.

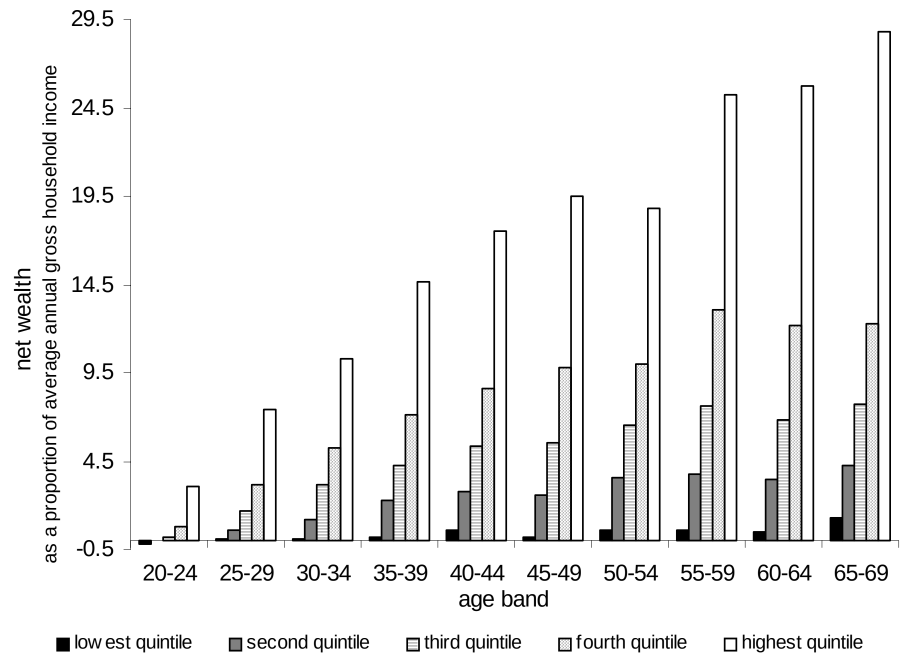
In addition to the statistics that are reported here, a number of recent studies have been conducted to describe the characteristics of UK households that tend to incur substantial debts. Bridges & Disney (2004), for example, use data from the Survey of Low Income Families to find that families with low incomes tend to incur more substantial arrears. They also find that arrears are positively correlated with the number of dependent children in a household, and negatively correlated with the age of the reference person (consistent with the statistics reported above). Similar observations were found by a Department of Trade and Industry report based upon the MORI Financial Services survey from 2004.[17] Kempson (2002) reports, using qualitative data from 1647 householders across Britain in 2002, that half of all households in financial difficulty attributed their problems to a loss of income, 15 percent to low income, and 10 percent to each of over-commitment, relationship break-down, and unanticipated

[16] 41% of households were identified as having at least one individual with an A-level qualification or higher.

[17] *Over-indebtedness in Britain*, op cit. In that report, individuals with low incomes, those with children and those in their 20s and 30s were found to be more commonly over-indebted than the population average.



Panel A: liquid wealth

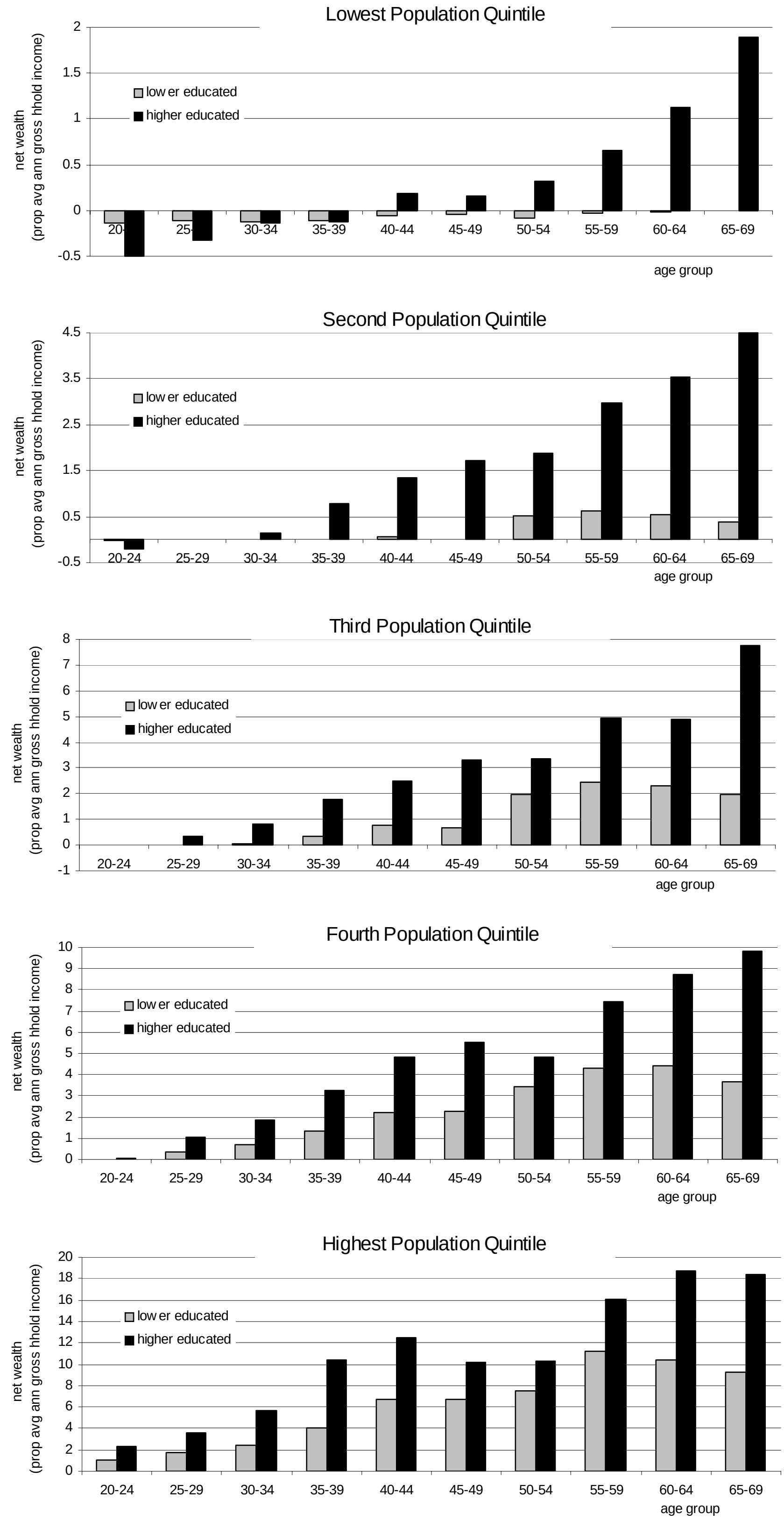


Panel B: total wealth

Source: Author Calculations using BHPS survey data

Average annual gross household income equal to £23,208.58. Figure calculated from (weighted) BHPS data for households 20-64 years of age in 2000/01

Figure 5: Average Net Wealth by Age Group and Wealth Quintile



Source: Author Calculations using BHPS survey data

Notes: Households identified as higher educated if any member recorded as having received an 'A-level' qualification or above

Average annual gross household income equal to £23,208.58. Figure calculated from (weighted) BHPS data for households 20-64 years of age in 2000/01

Figure 6: Average Net Liquid Wealth by Age Group, Education, and Wealth Quintile

expenses. For young people, low income was the main reason given for financial difficulties (accounting for 25 percent), followed by loss of income (23 percent), and over-commitment (14 percent).

Table 1 reports statistics for two regression models of household debt (based on liquid wealth) calculated using the BHPS survey data that are discussed above: a probit model, which describes the probability that a household is indebted given the various characteristics reported in the table; and a Tobit model, which indicates the magnitude of the associated debts.[18] The probit regression coefficients on age indicate that the probability of being indebted is particularly high between ages 20 and 30, and to a lesser extent between ages 30 and 40, after which the probability falls away strongly. Similarly, the Tobit model indicates that the size of debts held also falls strongly with age[19], following a period of variation between ages 20 and 40. These observations are clearly consistent with the distributional statistics discussed above.

Couples and divorcees have both a higher probability of being indebted, and larger debts, than do otherwise similar single adults. In the case of couples, this may reflect access to credit and/or expectations regarding future income prospects. In the case of divorce, it reflects the disruption that accompanies relationship breakdown, and is consistent with the findings reported by Kempson (2002) discussed above. In contrast to the various studies referred to above, however, the estimated statistics reported in Table 1 imply that households with dependent children tend to have a lower probability of being indebted, and are correlated with lower net debts. The differences between the statistics reported here and the previously mentioned studies are most likely attributable to the current focus upon net debt rather than arrears, and the fact that our sample is approximately nationally representative.[20]

All else being equal, education to A-level or above increases both the probability of being indebted, and the likely net debt if indebted, consistent with the statistics reported in Figure 6. The level of gross household income is associated with less pronounced indebtedness, whereas increases in income over the immediately preceding 12 month period are associated with higher indebtedness. Although this last observation may appear to contradict the associated findings reported by the studies that are referred to above, it is important to take expectations regarding future income into account. If an increase in income is considered to be transitory, then the life-cycle hypothesis implies that households should save some of the wind-fall gain to finance higher consumption in the future. This would imply

[18] The equations considered here were arrived at after experimenting with various alternatives. The estimated coefficients of the probit equation are not, in themselves, measures of probability. Nevertheless there is a positive association between the probit coefficients and the likelihood of a positive observation (e.g. an individual being indebted). Details regarding how probit coefficients translate into measures of probability are provided in any intermediate econometrics text book. See, for example, Johnston & DiNardo (1997), section 13.4.

[19] the Tobit regression is based upon an endogenous variable, where higher positive values indicate larger debts, and households with positive net assets are censored to zero.

[20] The possibility that the differences between the estimated effects of children on household indebtedness that are reported here, and those of the studies referred to above are attributable to omitted variable bias is unlikely, given that the estimates reported in Table 1 are robust to the alternative specifications that were explored. Importantly, the alternative specifications that were considered include employment identifiers, and a distinction between labour and non-labour income. These are omitted from the table because the associated regression coefficients were highly insignificant.

Table 1: Statistical Correlations between Debt and Household Characteristics – BHPS survey data

	Probit Model		Tobit Model	
Variable	*coefficient*	*std error*	*coefficient*	*std error*
Age				
20-24	0.17327	0.09641	-5959.29*	1925.54
25-29	0.12630	0.09520	-5150.32*	1937.56
30-34	0.02596	0.09641	-8540.21*	1988.32
35-39	0.03417	0.09881	-6805.88*	2030.71
40-44	-0.32548*	0.11042	-12047.40*	2289.51
45-49	-0.40931*	0.11554	-15655.60*	2426.10
50-54	-0.67024*	0.12325	-20342.10*	2625.29
55-59	-0.52917*	0.11621	-19326.70*	2476.65
60-64	-1.09780*	0.13609	-28590.40*	2979.80
65 and over	-1.77536*	0.08420	-40064.70*	2083.99
Demographic Status				
single parent	-0.01548	0.08813	-1795.74	1799.41
couple without children	0.38751*	0.07577	7130.05*	1562.04
couple with children	0.28779*	0.07902	3871.04*	1617.39
divorced	0.23240*	0.07732	3411.59*	1605.45
Education A-level or higher	9.04E-04	4.57E-03	2450.53*	1168.35
Gross household income	-4.64E-06*	2.66E-06	-0.16381*	0.05739
12 month change in income	3.07E-06	2.40E-06	0.09588	0.05088
House ownership	-1.73288*	0.06462	-27214.80*	1479.94
Log likelihood	-1567.94		-10879.1	
Observations	6260		6260	
Correct predictions (%)	87.939			
Sigma**			22062.80*	554.746

* estimate statistically significant at 95% confidence interval

** estimated standard deviation of residuals

Author calculations using data from waves 9 and 10 of the BHPS

Debt measured with regard to liquid wealth

a negative correlation between an income change and debt, consistent with the observations made by studies that explore the underlying reasons for over-indebtedness (including those discussed above). In contrast, if an increase in income is considered to indicate a larger increase in income in the future, then the life-cycle hypothesis will imply a positive correlation between income change and indebtedness, as observed here. This effect will be exaggerated when an increase in income also provides greater access to credit. Finally, the estimated statistics reported for house ownership in Table 1 indicate that residential property reduces both the probability of being indebted, and the size of net debts, consistent with the findings of earlier studies.

2.3 Summary

- Household sector indebtedness – both in absolute terms and relative to disposable income – has increased substantially during the last decade.
- The increased indebtedness of the household sector appears to be sustainable in context of the prevailing benign economic environment
- Household indebtedness appears to be heavily concentrated in the poorest fifth of young house-

holds (aged 30 and under). This has the advantage that young households have most of their working lives ahead of them, and have a longer period over which to repay the debt.

- Young individuals who are higher educated tend to be more heavily indebted than otherwise similar lower educated individuals. This has the advantage that higher educated individuals can be anticipated to be better able to afford the financial burden in the future.

- The rosy economic picture painted by population aggregate statistics masks severe financial hardship experienced by a minority of households who have experienced negative shocks to their circumstances, as in the case of employment disruption or divorce.

We now describe the rational agent model that is used to explore behaviour toward savings and indebtedness of heterogeneous agents during the life-course.

3 A Rational Agent Model of Consumption and Labour Supply

The decision unit in the model is the household. Households are considered to be fully described by ten characteristics:

age	- education	- number of adults	- number of children	- private pensions
wage rate	- labour supply	- consumption	- net liquid wealth	- time of death

The model is designed to consider household behaviour from age 20 to the maximum terminal age of 110, disaggregated into annual increments. Households choose their consumption and labour supply to maximise their expected lifetime utility, given their existing circumstances, preferences, and beliefs regarding the future. A household's circumstances are described by their age, education, number of adults, number of children, wage rate, net liquid (non-pension) wealth, private pension rights, and time of death. Preferences are defined by a utility function (that is the same for all households), and beliefs are rational in the sense that they reflect the processes that generate household circumstances.

Care has been taken in specifying the model to capture factors that are important in determining the financial welfare of households and their use of debt. The preceding section indicates that age, education, and the treatment of pension wealth have an important bearing on the extent of indebtedness that is identified by survey data for the population. Furthermore, incorporating an appreciation of uncertainty into individual expectations regarding future circumstances dramatically increases the complexity of the utility maximisation problem. As noted in the preceding section, the most prevalent causes given for financial hardship are the loss of income, low income / over commitment, relationship breakdown, and unanticipated expenses. Hence, of the eight characteristics that define the circumstances of a household, four are considered to be stochastic (labour income, relationship status, number of children, and time of death), and four are considered to be deterministic (age, education, private pensions and net liquid wealth).

In the terminology of the dynamic programming literature, consumption and labour supply are control variables that are considered to be selected to maximise the value function described by a time separable utility function, subject to eight state variables, four of which are stochastic, and four deterministic. Disaggregation of liquid wealth into housing and non-housing wealth, and identifying collateralised debt, are undertaken outside of the formal dynamic programming problem.

Outline

This section begins by defining the assumed preference relation, before describing the wealth constraint, the processes assumed for the evolution of income and household size, and concludes with an explanation of the approach adopted to solve the lifetime utility maximisation problem. Calibration of the model is reported in Section 4, and the methods adopted to simulate housing wealth and collateralised debt are described in Section 5. The non-technical reader may skip directly to the discussion of the intuition behind the life-cycle model provided in Section 6 without excessive handicap.

3.1 The utility function

Expected lifetime utility of household i at age t is described by the time separable function:

$$U_{i,t} = \frac{1}{(1-1/\gamma)} E_t \sum_{j=t}^{110} u\left(\frac{c_{i,j}}{\theta_{i,j}}, l_{i,j}\right)^{1-1/\gamma} \delta^{j-t} \phi_{j-t,t} \tag{1}$$

where $\gamma > 0$ is the intertemporal elasticity of substitution (of total expenditure), E_t is the expectations operator, $c_{i,t} \in R^+$ is composite nondurable consumption, $l_{i,t} \in \{l_{W,na}, 1\}$ is the proportion of household time spent in leisure, and $\theta_{i,t} \in R^+$ is adult equivalent size based upon the McClements' scale.[21] From age 65, the household is forced to retire if it has not already chosen to do so, in which case $l_{i,t} = 1$ for all $t \geq t_{SPA} = 65$.

The labour supply decision is considered to be made between discrete alternatives, in response to the view that this provides a closer reflection of reality than one in which labour supply is a continuous choice variable for given wage rates.[22] It also reflects the current focus on household indebtedness, and

[21] See McClements (1977) on the McClements' equivalence scale, and Balcer & Sadka (1986) and Muellbauer & van de Ven (2004) on the use of this form of adjustment for household size in the utility function.

[22] Although working time arrangements have become increasingly flexible since the 1970s, substantial labour market rigidities continue to affect employment decisions. Fagan (2003), for example, reports that approximately 1 in 5 employed people in Europe work full-time when they would prefer to work part-time. The reasons most commonly given for the mis-match include the perception that it would not be possible to do a desired job part-time, that part-time employment is not offered by a desired employer, and that it would damage career prospects. Furthermore, cross-sectional data from the 2002/03 Family Resources Survey indicate that approximately 80 percent of household income is earned by the household reference person between ages 20 and 65 – a proportion that is approximately stable after age 27. The correlation between the proportion of household reference people employed full-time by age, and the proportion participating in the labour force is 0.971 between ages 20 and 65, and 0.978 for the population more generally. Higher correlations are observed between ages 55 and 64. The issue of heterogeneity of employment decisions is further complicated in the current context by the considered policy environment (which tends to discourage part-time employment in later life due to the influence of means tested benefits).

our view that this is influenced more by the extensive than the intensive labour margin. It should be noted, however, that the assumption of a dichotomous labour supply decision is likely to dampen the responsiveness of labour supply behaviour implied by the simulation model. Calibration of the model is consequently likely to require a labour elasticity that overstates the practical reality. Furthermore, omitting heterogeneity in the labour supply decision is likely to imply that the calibrated income process will involve more variation than applies in practice.

The McClements' scale depends upon the numbers of adults, $n^a_{i,t}$, and children, $n^c_{i,t}$ in a household, and its inclusion in the preference relation reflects the fact that household size has been found to have an important influence on the timing of consumption (e.g. Attanasio & Weber (1995) and Blundell et al. (1994)).[23] $\phi_{j-t,t}$ is the probability of living to age j, given survival to age t, and δ is the discount factor, which is assumed to be the same for all households and time invariant.[24]

A Constant Elasticity of Substitution function was selected for within period utility,

$$u\left(\frac{c_{i,j}}{\theta_{i,j}}, l_{i,t}\right) = \left(\left(\frac{c_{i,j}}{\theta_{i,j}}\right)^{(1-1/\varepsilon)} + \alpha^{1/\varepsilon} l_{i,t}^{(1-1/\varepsilon)}\right)^{\frac{1}{1-1/\varepsilon}} \tag{2}$$

where $\varepsilon > 0$ is the (period specific) elasticity of substitution between equivalised consumption $(c_{i,t}/\theta_{i,t})$ and leisure $(l_{i,t})$. The constant $\alpha > 0$ is referred to as the utility price of leisure. The specification of intertemporal preferences described by equations (1) and (2) is standard in the literature, despite the contention associated with the assumption of time separability (see Deaton & Muellbauer (1980), pp. 124-125, or Hicks (1939), p. 261). The specification of equation (2) implicitly assumes that characteristics which affect utility, but are not explicitly stated, enter the utility function in an additive way.

3.2 The wealth constraint

Equation (1) is considered to be maximised, subject to an age specific credit constraint imposed on liquid net worth, $w_{i,t} \geq D_t$. We define liquid net worth by:

$$w_{i,t} = w_{i,t-1} + \tau\left(l_{i,t-1}, r_{i,t-1}w_{i,t-1} + x_{i,t-1}, n^a_{i,t-1}, n^c_{i,t-1}, t-1\right) - c_{i,t-1} \tag{3}$$

where $r_{i,t} = r$ is the real interest rate (considered to be time invariant and the same for all individuals), $x_{i,t}$ is private non-property income, and $\tau(.)$ is the tax and benefit function. In practice, liquid net worth, $w_{i,t}$, is comprised of housing, cash balances, risky financial investments, and so on. Demand for these alternative asset classes is affected by a range of considerations, including the associated transactions

[23] An empirical study by Fernandez-Villaverde & Krueger (2006) of US data from the Consumer Expenditure Survey suggests that roughly half of the variation observed for lifetime household consumption can be explained by changes in household size, as described by equivalence scales.

[24] See, for example, Gustman & Steinmeier (2005) for a structural analysis of household savings and retirement decisions that allows for a varying discount factor between households.

costs, the uncertainty of investment returns, differential tax treatment, and the consumption of housing services. We simplify the analysis by abstracting from the asset allocation problem, and leave associated sensitivity analysis as an issue for further research.

The function τ is a stylised representation of the UK tax and benefit system, and is described in detail in subsection 3.3. During the working lifetime, $t < t_{SPA}$, $x_{i,t}$ defines household labour income, equal to the wage rate $h_{i,t}$ multiplied by a random factor $\varphi_{i,t}$ if the household chooses to work, and zero otherwise. The random factor is a dummy variable that identifies when a household receives a wage offer (and thereby controls for involuntary unemployment). Importantly, the probability of a low wage offer is considered to depend only upon marital status and is independent of age, so that the profile of rates of employment is shaped by preferences and rates of pay, rather than by the exogenously defined probability of a low wage offer. The household wage, $h_{i,t}$, is considered to evolve following a stochastic process that is described in subsection 3.4.

During retirement, $t \geq t_{SPA}$, $x_{i,t}$ is equal to the annuity income generated by private pensions, $op_{i,t_{SPA}}$:[25]

$$x_{i,t} = \begin{cases} \varphi_{i,t} h_{i,t} & \text{if } t < t_{SPA} \\ op_{i,t_{SPA}} & \text{if } t = t_{SPA} \\ x_{i,t-1} \frac{\left(0.5+0.5\left(n^a_{i,t}-1\right)\right)}{\left(0.5+0.5\left(n^a_{i,t-1}-1\right)\right)} & \text{if } t > t_{SPA} \end{cases} \tag{4}$$

As discussed at length in Section 2, the extent of household net indebtedness varies greatly, depending upon the treatment of occupational and personal (private) pensions. We model private pensions explicitly, by assuming that all employed individuals accrue private pension rights according to a specification that is similar to the process of rights accrual for the State Earnings Related Pension Scheme (SERPS). SERPS was applied between 1978 and 2002, and imposed a stronger link between contributions and pension rights than its successor, the State Second Pension (S2P). As such, SERPS provides a closer approximation to occupational and personal pensions than S2P. Furthermore, to make the rights accrual that was applied under SERPS more similar to private pensions, we omit the Upper Earnings Limit on incomes that was imposed by SERPS. Although our adoption of a defined contribution framework for private pensions, rather than a defined benefits framework, is at odds with the historical norm, it does reflect the strong prevailing trend in occupational pensions provision.

We consequently assume that private pension rights are accrued according to:

$$op_{i,t_{SPA}} = \sum_{t=20}^{t_{SPA}} \frac{\Psi \rho^{(t_{SPA}-t)} \max\{0, x_{i,t} - LEL\}}{[t_{SPA} - 19]} \tag{5}$$

where $\rho = 101.9\%$ is the rate of adjustment for real wage growth, $LEL = 77$ is the Lower Earnings Limit

[25] The annuity purchased at age t_{spa} is assumed to reduce to 65% when the number of adults in a simulated household decreases to 1 in response to the mortality of a spouse. This adjustment to retirement income was necessary to capture the decline in expenditure with age observed in survey data.

as applied in 2003, and $\Psi = 0.25$ is the accrual rate that was applied by SERPS for the 1978/79-1987/88 tax years (the most generous rate applied by the scheme).[26]

The interest rate is assumed to depend upon whether $w_{i,t}$ indicates net investment assets, or net debts:

$$r_{i,t} = \begin{cases} r^I & \text{if } w_{i,t} > 0 \\ r_l^D + \left(r_u^D - r_l^D\right) \min\left\{\frac{-w_{i,t}}{\max[x_{i,t}, 0.7h_{i,t}]}, 1\right\}, r_l^D < r_u^D & \text{if } w_{i,t} \leq 0 \end{cases}$$

This specification for the interest rate implies that the interest charge on debt increases from a minimum of r_l^D when the debt to income ratio is low, up to a maximum rate of r_u^D, when the ratio is high. The specification also means that households in debt are treated more generously if they are working than if they are not. The assumption that the maximum rate of interest is charged when net debt is equal to or greater than the household full-time employment wage reflects the observation that less than 1 percent of households recorded by the 2000/01 BHPS with some labour income had unsecured debt that exceeded their annual gross labour income.

3.3 The tax function

The function τ is a stylised representation of the UK tax and benefits system, described as a function of the household's pre-tax income, that is its property income $r_{i,t}w_{i,t}$ plus non-property income $x_{i,t}$, its size $n_{i,t}^a$ and $n_{i,t}^c$, and its age, t. The age dependency assumed for the tax function divides the lifetime into three periods: the working lifetime $t < t_{IB} = 55$, early retirement $t_{IB} \leq t < t_{SPA} = 65$, and retirement $t_{SPA} \leq t$. During the working lifetime, the tax function is specified to reflect profiles reported in the April 2003 edition of the *Tax Benefit Model Tables* (TBMT) issued by the Department for Work and Pensions.[27] The profiles considered take into consideration the impact of income taxes, National Insurance Contributions, the Child Benefit, the Working Tax Credit and the Child Tax Credit. Although this list omits a great deal of the detail of the UK tax and benefits system, it does include the principal schemes that affected healthy families with children during 2003.[28] Furthermore, the tax function includes an increase in the marginal tax rate on earnings, to account for contributions to private pensions.[29]

[26] On the basis of contributions equal to 4.4% of gross employment income (equal to the contracted out rebate on SERPS), this pension scheme implies a real internal rate of return of 4.2% per annum (very close to the 4.0% rate of return that is considered for liquid assets in the model calibration) and a replacement rate (of gross income at age 65) of 36%, based on the age profile of geometric mean disposable income for all households.

[27] See http://www.dwp.gov.uk/asd/tbmt.asp.

[28] The focus on a single labour supply term for households raises complications for the tax function that is considered for couples. The UK tax system is based upon individual incomes – a couple cannot split their income to minimise their aggregate tax burden. The simulation of household income, as opposed to individual specific income, implies that some allowance could be made to take into account the tax effect of dual income households. Data from the 2002/03 FRS indicate that, on average, 80 percent of labour income earned by couples is attributable to the principal bread winner between ages 20 and 64 (the proportion is slightly lower at 76 percent between 20 and 30, and slightly higher after age 60 at 85 percent). Given this observation, we assume that all income is earned by the principal bread winner, and acknowledge that this will slightly overstate the true tax burden faced by dual income households.

[29] The marginal tax rate is increased by 4.4% on earnings in excess of the Lower Earnings Limit, as it stood in 2003 (£77 per week). This is based upon the contracted out rebate for employer contributions, scaled up by 25% to reflect the fact that we adopt the most generous accrual rate that was applied at any time by SERPS.

The simulated tax function for ages $t_{IB} \leq t < t_{spa}$ depends upon private income, employment status, age, and demographic composition. Simulated households that choose to supply labour for any t, $t_{IB} \leq t < t_{spa}$, are treated in the same way as during the working lifetime (described above). The tax treatment applied to a simulated household that chooses not to supply labour and is aged $t_{IB} \leq t < t_{MIG} = 60$, is specified to reflect the Incapacity Benefit and income taxes as they stood in 2003/4; between ages $t_{MIG} \leq t < t_{spa}$ the tax function is specified to reflect the Pension Guarantee and income taxes.

The specification of the tax function during retirement, $\tau(.)$, $t \geq t_{spa}$, is based upon the Basic State Pension (which is assumed to be received by every household), and the Pension Credit as they were applied in the UK during 2003.

Further details of the tax system can be found in Appendix B.

3.4 Household income dynamics

In the first period of the simulated lifetime, age 20, each household is allocated a wage, $h_{i,20}$, via a random draw from a log-normal distribution, $\log(h_{i,20}) \sim N\left(\mu_{na,20}, \sigma^2_{na,20}\right)$, where the parameters of the distribution depend upon the number of adults in the household, na. Thereafter, wages are generated using the stochastic process described by the equation:

$$\log\left(\frac{h_{i,t}}{m\left(na_{i,t}, t\right)}\right) = \beta_{na_{i,t-1}} \log\left(\frac{h_{i,t-1}}{m\left(na_{i,t-1}, t-1\right)}\right) + \omega_{i,t} \tag{6}$$

where $m(.)$ is a function that accounts for wage growth and depends on age, t, and the number of adults in the household, $na_{i,t}$, $\beta_{na_{i,t}}$ accounts for time persistence in earnings, and $\omega_{i,t} \sim N\left(0, \sigma^2_{\omega,na_{i,t-1}}\right)$ is a household specific disturbance term. A change in the number of adults in a household affects wages through the persistence term, β, and the wage growth function $m(.)$. This model is closely related to alternatives that have been developed in the literature (see Sefton & van de Ven (2004) for discussion), and has the practical advantage that it depends only upon variables from the current and immediately preceding periods $(t-1, na_{i,t-1}, na_{i,t}, h_{i,t-1}, l_{i,t-1})$, which simplifies the endogenous simulation of household savings and labour supply.

3.5 Modelling adults and children

The numbers of adults and children in a household are considered to develop in a probabilistic fashion, following a (reduced form) nested logit model. The model is comprised of two levels, where the first (highest) determines the evolution of the number of adults in a household, and the second determines the number of children, given the evolution considered for the number of adults and age.

A household can be comprised of one or two adults between ages 20 and 89, where the number of adults is considered to be uncertain between adjacent years. From age 90, all households are comprised of a single adult. The fact that children typically remain dependants in a household for a limited number of years implies that it is necessary to record both their numbers and age when including them in the rational agent model, which substantially increases the computational burden. If, for example, a household was considered to be able to have children at any age between 20 and 45, with no more than one birth in any year, and no more than six dependent children at any one time, then this would add an additional 334,622 state variables to the computation problem (with a proportional increase in the associated computation time). It was consequently necessary to restrict the manner in which children are included in the model.

We assume that a household can receive up to three children at two discrete ages, so that the maximum number of dependent children in a household at any one time is limited to six. The two transition ages, and the upper limits assumed for the number of children that can be "born" at each age, were selected with reference to survey data reported in the BHPS. Figure 7 reports the cumulative proportion of births by household age, calculated from wave 7 of the BHPS (the middle wave of the full panel that is currently available). Four key ages are singled out in the figure, approximately corresponding to cumulative aggregates of 25%, 50%, 75%, and 95% of births. The ages at which a household is considered to potentially receive children are located at the 25% and 75% thresholds – ages 25 and 34. The ages selected for the calculating the regression of the ordered logit models for the numbers of children borne at ages 25 and 34 relate, respectively, to the 50% and 95% thresholds displayed in Figure 7. Furthermore, in the case of the upper limits imposed on the number of children that can be borne into a household at each age, 2.6% of households in the BHPS data pooled over all 13 waves of the survey have more than three children by age 25, and less than 1.0% have more than three children younger than 11 by age 41.

The logit model considered to describe the evolution of adults in a household is described by equation (7):

$$s_{i,t+1} = \alpha_0^A + \alpha_1^A t + \alpha_2^A t^2 + \alpha_3^A t^3 + \alpha_4^A dk_{i,t} + \alpha_5^A s_{i,t} \tag{7}$$

where $s_{i,t}$ is a dummy variable, that takes the value 1 if household i is comprised of a single adult at age t and zero otherwise, and $dk_{i,t}$ is a dummy variable that equals 1 if household i at age t has at least one child. With regard to the simulation of births, four separate ordered logit equations were estimated; one for each of single and couple households, at ages 30 and 41. The ordered logit equations assumed for child birth at age 30 for both singles and couples do not include any additional household characteristics, so that the logit estimation involved regressing the dependent variable (the number of children aged 17 or under for singles or couple households aged 30) against a constant. The associated regression statistics consequently describe the simple proportions of single or couple households identified in the

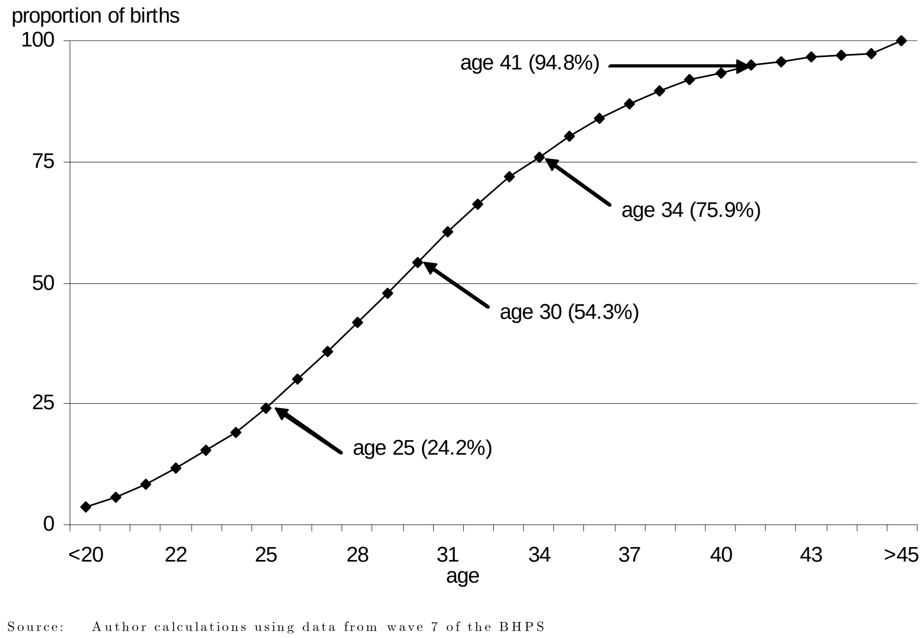


Figure 7: Cumulative Proportion of Births by Household Age

survey data with 0, 1, 2 or 3 or more children respectively. The ordered logit equations for child birth at age 41 include the number of children aged 11 or over as an additional descriptive characteristic.

The logit equations described above were estimated separately using pooled data derived from waves 1 (1990/91) to 13 (2004/05) of the BHPS, the most recently published data at the time of writing.[30] Limiting the BHPS sample to omit households for which incomplete information was available for any of the relevant characteristics, or households in which the reference person was under the age of 20, resulted in a total sample size of 65306 observations used for estimation. Furthermore, the sample was divided by education status, based upon the highest academic qualification of the household, distinguishing those with 'A level' qualifications or above from those without (as in Section 2.2).

Regression statistics are reported in Table 2.

The regression statistics reported in the top panel of Table 2 imply intuitive dynamics for the number of adults in a household.[31] The coefficients on age suggest that the proportion of single adult households is low and approximately stable until age 55, when it starts to rise in response to spousal mortality. The presence of dependent children tend to make it less likely that a household will be comprised of a single adult. Furthermore, a household is more likely to be single at any given age, if it was single the

[30] Separate regressions were calculated for each of the equations, rather than estimating a single Multinomial Logit model, to allow for the likely violation by the model of the Independence of Irrelevant Alternatives. See, for example, Chapter 19 of Greene (1997) for details.

[31] Estimations for a logit model provide a measure of the frequency of a given characteristic (in our case the prevalence of couples, for example), relative to other observed characteristics described by a sample data set. Like a probit regression, the estimated coefficients of a logit model are not measures of probability, but are positively related to associated probabilities. For further details, see for example, Johnston & DiNardo (1997), section 13.4.

Table 2: Regression Statistics for Nested Logit Model of Household Size

	full population		lower educated		higher educated	
Variable	*coefficient*	*std. error*	*coefficient*	*std. error*	*coefficient*	*std. error*
	Single in period t+1					
constant	-4.54486	0.40648	-3.63606	0.56777	-6.31619	0.62662
age	9.51E-02	2.60E-02	3.46E-02*	3.52E-02	2.31E-01	4.19E-02
age^2	-2.20E-03	5.12E-04	-1.07E-03*	6.79E-04	-4.99E-03	8.63E-04
age^3	1.75E-05	3.16E-06	1.10E-05	4.11E-06	3.44E-05	5.54E-06
children	-1.02841	0.04853	-1.09673	0.07549	-1.16721	0.06747
single	5.58329	0.03734	5.89854	0.05223	5.05855	0.05514
correct predictions	0.94654		0.94841		0.94404	
sample	65306		37394		27912	
proportion single	0.32979		0.42095		0.20765	
	number of children - single adults aged 30					
MU(child=1)	0.35364	0.11356	-0.35937	0.15148	1.47810	0.21733
MU(child=2)	1.04922	0.12754	0.45199	0.15289	2.27924	0.29120
MU(child=3)	2.30603	0.19473	1.73460	0.20874	4.23411	0.71221
sample	320		180		140	
	number of children - couples aged 30					
MU(child=1)	-0.38274	0.06874	-1.21487	0.13328	0.02505	0.08460
MU(child=2)	0.54132	0.06998	-0.36124	0.11381	1.14446	0.09882
MU(child=3)	1.99632	0.10401	1.19719	0.13265	2.83511	0.18480
sample	878		319		559	
	number of children between ages 31-41 - single adults					
children at age 30	0.52132	0.18456	0.31047	0.22383	0.93588	0.35004
MU(child=1)	2.08936	0.20790	1.79826	0.26189	2.50353	0.35024
MU(child=2)	3.21106	0.29491	2.91657	0.36381	3.65458	0.51030
MU(child=3)	5.27710	0.72166	5.27967	1.01624	5.30611	1.03151
sample	293		162		131	
	number of children between ages 31-41 - couples					
children at age 30	-0.6055518	0.068916	-0.3280282	0.1017836	-0.7932152	0.0956868
MU(child=1)	-0.7767481	0.0860932	-0.3582378	0.1394233	-1.021973	0.111479
MU(child=2)	0.6196216	0.0848925	1.05466	0.1494134	0.3959916	0.1040875
MU(child=3)	2.442878	0.1382957	2.90745	0.2594715	2.230084	0.1637099
sample	1053		423		630	

Author calculations based on pooled data from waves 1 to 13 of the BHPS

MU(child=z) lower logit threshold for z number of children borne at relevant age

* not significant at 95% confidence interval

probabilities implied by the above coefficients are equal to 1/(1+exp(-y)), where y is the dependant variable year before.[32]

The estimates reported in Table 2 also indicate that couples tend to have more children than do single adults at both transition ages. If a child was borne at the younger transition age, then the probability of a birth at the older child bearing age is higher for singles, and lower probability for couples. Furthermore, lower educated households tend to have more children than higher educated, except for couples at the older child bearing age.

3.6 Model solution procedure

This subsection provides a brief summary of the model solution procedure; see Sefton (2000) for a detailed description.

The assumption of stochastic income implies that an analytical solution to the utility maximisation problem does not exist. The procedure that we adopt consequently uses backward induction to solve the required inter-temporal Bellman equation. Starting in the last possible period of the household's life, T (= 110 in the simulations), we can solve relatively simply for the optimising consumption behaviour, given wealth w_T and annuity h_T (where we redefine h_t to denote annuity income for all $t \geq t_{SPA}$, and the household wage otherwise).[33] Given this level of consumption we can denote the maximum achievable utility, the value function, by $V_T(w_T, h_T)$. We calculate this function at all nodes of a two dimensional grid in wealth and retirement annuity.

At time $T-1$ the problem reduces to solving the Bellman equation:

$$V_{T-1}(w_{T-1}, h_{T-1}) = \max_{c_{T-1}, l_{T-1}} \left(u\left(c_{T-1}/\theta_{T-1}, l_{T-1}\right) + \delta\phi_{1,T-1} E_{T-1}\left(V_T(w_{T,}, h_T)\right)\right) \tag{8}$$

We solve this optimisation problem for each node of the $T-1$ value function grid. This process is repeated to obtain successive solutions by backward induction. Post mandatory retirement (periods $65 \leq t \leq T$), this implies searching over feasible consumption choices only. Prior to mandatory retirement (periods $20 \leq t < 65$), we need to search over the feasible consumption choices for each of the discrete choices of labour supply considered for analysis, and then select the particular consumption/leisure pair that achieves the maximum utility. Furthermore, during the working lifetime, future wages are uncertain, and subject to a log-normal distribution. In this case, expectations of next period's value function are evaluated using a gaussian quadrature procedure with 5 abscissae points. A linear interpolation procedure is used to evaluate the value function at points between nodes throughout the simulated lifetime.[34]

[32] Including a lagged dependent variable in the regression allows the specification to be redefined in terms of the observed change in marital status.

[33] The version of the model considered here does not include an explicit bequest motive (though accidental bequests are generated). Hence, in period T, households choose to consume all remaining resources.

[34] With regard to computation time, there is a trade-off between the grid resolution and the order of the interpolation procedure. As the value function is not globally smooth or concave, we achieved greater accuracy by adopting a fine grid,

For the analysis reported in Section 7, we divided the real non-negative domain of both private income and wealth using a log scale, each comprised of 61 points from ages 20 to 64, and 151 points during each of the periods from age 65. From age 20 to 89 (inclusive), solutions are required for single adults and couples. Between ages 25 and 42, solutions are required for 0 to 3 children, and for an additional 0 to 3 children between ages 34 and 51. Between ages 20 and 64, solutions are required for a low wage offer on the one hand (in which case households are considered to choose not to supply labour), and the two labour options on the other. Furthermore, between ages 20 and 64 the dimension recording a household's private pension entitlement was divided into 11 points. Hence, the full model involves calculating solutions to expected lifetime utility maximisation problems at 59,085,995 points for the simulated lifetime.[35]

Having solved for the household utility maximising behavioural responses as described above, the life-courses of individual households are simulated by running households forward through the grids. For example, given a household's initial wealth, wage offer, and number of adults $(w_{20}, \varphi_{20}, h_{20}, n_{20}^a)$ we read off from the age 20 grid the household's optimal choice of consumption and leisure (c_{20}, l_{20}). Then three random draws are taken: one from a uniform distribution that is compared against the probability implied by equation (7) to determine the number of adults at age 21, n_{21}^a; another from a uniform distribution to determine whether a high wage offer is made φ_{21}; and the third from the distribution $\omega_{i,t} \sim N\left(0, \sigma_\omega^2\right)$, to obtain the household wage rate at age 21 described by equation (6), h_{21}. Equation (3) is also used to calculate the household's wealth, so that all four state variables for age 21 are determined, $(w_{21}, \varphi_{21}, h_{21}, n_{21}^a)$. This process is repeated for $t = 21, 22, ...T$. A cohort is built up by repeating this procedure for a sample of households. Our analysis is based on the data generated for this synthetic population. We now discuss some technical issues that arise in the optimisation procedure.

The value function in this problem is neither smooth, nor concave (though it is increasing and continuous). This is because the labour supply decision is discrete and the budget set is non-convex (due to means testing of welfare benefits). Non-concavities of the value function imply that the optimisation problem (8) can have local maxima, a problem that can be addressed in a number of alternative ways. French (2005), for example, adopts a brute-force method by searching across the grid that defines all possible choice combinations of the assumed control variables. This is very time consuming, which

and a linear interpolation routine. Both the wealth and wage grid dimensions were specified on a log scale (to provide greater detail at the low end of the distribution).

[35] =151*151*((110-89)+2*(89-64))+61*61*2*3*11*((64-51)+4*(51-42)+4*4*(42-33)+4*(33-24)+(24-19)) Tests were conducted with a finer grid structure, based upon 101 points for each of the wealth and private income dimensions, to which the simulated behaviour was insensitive. This full model required just over 17 hours to run on an Intel X6800 processor. Most of the runs undertaken to calibrate the model were undertaken without concavity checks, by adopting a course grid structure, and by omitting pension wealth (but assuming forced annuitisation at age 65). These simplifications reduced the model run time to just under 15 minutes on a Pentium D processor.

complicates calibration and associated analysis. We therefore chose to solve the Euler equations (for consumption), and guard against the selection of a local optimum. This was relatively straightforward for our decision problem, as we were searching over only one continuous variable (consumption). The procedure that we adopt is described as follows.

For a given discrete labour supply choice, we searched for a solution to the Euler equation over all feasible (bounded) consumption choices using the Bus & Dekker (1975) bisection algorithm. Having found one solution, we searched above and below for an alternative solution.[36] If we found one, we searched recursively for any further solutions above and below. This was repeated until all solutions were found. From these solutions we selected only those that were feasible.[37] Of all feasible solutions, the one that maximised the value function was selected. This procedure was repeated for all possible discrete labour choices, and the maximum over the labour choices was selected.

3.7 Summary

- The rational agent model considers household decisions regarding consumption and labour supply at annual intervals throughout a life course that extends from age 20 to the maximum potential age of 110.
- Household heterogeneity is described by eight characteristics: age, education, *number of adults*, *number of children*, *potential wage*, net liquid wealth, pension rights, *time of death*; where the characteristics in italics are generated stochastically.
- Intertemporal preferences are time separable, and take a Constant Relative Risk Aversion (CRRA) form.
- Intratemporal preferences between consumption and leisure take a Constant Elasticity of Substitution (CES) form.
- Expected lifetime utility is considered to be maximised, subject to an age specific liquidity constraint that permits the holding of unsecured debt.
- The interest charged on unsecured debt is an increasing function of a household's debt to income ratio.
- Taxes and benefits are specified to reflect the UK transfer system as it stood in 2003.

[36] This involved projecting a limited grid about an identified local maximum, and evaluating the first order condition for the Euler maximisation problem at each point on the limited grid. If the first order condition at a point on the limited grid below (above) the initial local maximum was found to have the opposite sign to it's value just slightly below (above) the originally identified local maximum, then the bisection algorithm was reapplied to solve the associated Euler conditions for a new local maximum.

[37] This was a check that marginal change in the value function with respect to wealth was locally increasing. Non-smoothness in the value function implies jumps in marginal rates. The interpolation routines will tend to smooth these jumps, possibly introducing an infeasible solution to the Euler equation.

- Income dynamics are modelled by a regression toward the mean process.
- The model is solved numerically by backward induction.

We now describe the methods used to calibrate the parameters of the model.

4 Model Calibration

The parameters of the model were adjusted to match the characteristics by age of a simulated population to those described by household micro data. Calibration was undertaken using the following gird-search procedure. First, wealth at age 20 was set to 0. $l_{W,na}$ was set to 0.5 for both couples and singles (working entails a 50% reduction in leisure). We then normalised by the price of consumption so that wages and interest rates were specified in real terms. The real interest rate was fixed at 4 percent per annum for non-negative wealth balances $\left(r^I\right)$, the lower limit cost of debt $\left(r_l^D\right)$ was set to 7 percent, and the upper limit $\left(r_u^D\right)$ to 15 percent. This range of interest charges for household debt is based upon averages between January 2000 and August 2005 (the most recently available data at the time of writing) of the end of month average interest rates charged on unsecured personal loans for the lower bound, and of the end of month weighted average interest rates charged on credit cards for the upper bound, as reported by the Bank of England.[38]

The credit limit, D_t, that is considered for analysis is specified to provide maximum flexibility, while at the same time ensuring that households have no debt from age 65, and omitting the possibility of numerical errors in the solution procedure. A numerical error will result if a household is required to consume less than or equal to zero in any period to satisfy the imposed budget constraint. This, combined with the assumption that all households must have paid off their debts by age 65, resulted in the adoption of an age specific credit limit that is defined in terms of the minimum disposable income stream that is permissable under the analysis.

In defining the assumed credit limit, we took the lowest potential disposable annual income at any age (as described by the considered tax and benefits system), deducted a £5 disregard, and calculated the cumulative discounted value to age 65 implied by the maximum assumed interest charge on debt. The resultant age specific credit limits are reported in Table 4, and indicate that the assumed credit limit is over £20,000 for most of the working lifetime. To help identify how the assumed credit limit relates to the actual circumstances of households in the UK, Table 3 reports distributional statistics for unsecured debt identified by four recent surveys.[39] Table 3 indicates that most households that take

[38] The average between January 2000 and June 2007 of the end of month interest rate charged on unsecured personal loans reported by the Bank of England (code IUMHPTL), is 9.3 percent, and for credit card debt (code IUMCCTL) is 16.5 percent. Discounting by the Consumer Prices Index (ONS code D7G7) for the same period, gives real interest rates of 7.6 and 14.7 percent respectively.

[39] The surveys include a study commissioned by the Bank of England (Tudela & Young (2003)), a study commissioned by the CAB (2003), a study by KPMG (2003), and a study by Kempson (2002).

Table 3: Distribution of the Amount Owed by Debtors (Unsecured Debt)

Source: Table D, Tudela & Young (2003)

out unsecured loans in the UK borrow small amounts. Importantly, no more than 2% of households interviewed on behalf of the Bank of England or the Citizens' Advice Bureaux, and 7% of households interviewed for KPMG, held unsecured debts totalling £20,000 or more.[40]

Having defined the model parameters referred to above, we then selected a starting value for each of the remaining parameters, against which a solution to the lifetime optimisation problem was obtained following the methods described in Section 3.6. Monte-Carlo methods were used to generate the life-history for a cohort of households, based upon the behavioural responses described by the model solution, and the stochastic processes assumed for the intertemporal development of agent specific state variables. Calibration proceeded by comparing the characteristics by age of the simulated cohort with age profiles that were estimated from survey data.

The model was calibrated three times: first against age profiles estimated from survey data for a representative sample of the entire UK population, and then against age profiles estimated for lower educated households, and finally for higher educated households. The first calibration involved matching the model to distributional statistics for labour supply, disposable non-property income, and consumption. For the education specific subgroups, only the income parameters of the model were altered, so that all of the simulations reported in Sections 7 and 8 are based upon the same (unobservable) preference parameters. Furthermore, due to the absence of required survey data, none of the calibrations are matched directly to measures of household wealth or debt. This is of note, given our focus on household indebtedness.

Outline

The section begins by describing the sample statistics against which the model was calibrated. It then discusses issues related to calibration of the income processes and preference parameters, before reporting the calibrated parameters. The age profiles against which the model was calibrated are

[40] More recently, 5% of debtors reported debts in excess of £20,000 to a survey undertaken in September 2006 by NMG Research for the Bank of England. See Waldron & Young (2006), p. 398.

reported alongside the calibrated parameters for the calibration to the representative sample of the full UK population, and for the education subgroups in Appendix C.

4.1 The data

To calibrate our model we require age profiles at the household level for both singles and couples of[41]:

1. The proportion of households with at least one member employed, by education status
2. The mean of household private non-property income (gross of taxes and benefits), by education status
3. The variance of household private non-property income, by education status
4. The mean of household disposable income (net of taxes and benefits)
5. The variance of household disposable income
6. The mean of household total consumption
7. The variance of household total consumption

The parameters of models similar to the type considered here are usually matched against moments estimated from one of three survey data sources. The first is (pseudo) panel data for an actual cohort. These data are affected by time and cohort effects that make them unrepresentative for the population in general.[42] Alternatively, it is possible to control for time and cohort effects by econometric estimation. In this case, collinearity between age, cohort and time implies the need to introduce an additional restriction to permit identification. One popular restriction, suggested by Deaton (1997), is to assume that time effects average out over the long run. This assumption produces estimated age profiles that represent an average taken over all cohorts included in the (pseudo) panel data set used for estimation. Calibrating the model against this sort of age profile implicitly assumes that behaviour is invariant to changes in the policy environment during the period of estimation, which is an assumption that is difficult to maintain when one of the objectives of the analysis is to consider behavioural responses to alternative policy experiments.[43] The third approach involves matching the model against suitably adjusted age profiles described by cross-sectional survey data.

We adopted the third approach referred to above for the study reported here. The cross-sectional data considered for the calibrations describe the decisions of agents given their prevailing circumstances.

[41] Statistics of household wealth are prominently omitted from this list due to the limited nature of wealth data that are currently available for the UK. Nevertheless, a "sanity check" was performed by comparing age averages for wealth generated by the model against sample statistics imputed from the 2000/01 wave of the BHPS. These are available from the author upon request.

[42] See, for example, Attanasio et al. (2005).

[43] See, for example, Sefton et al. (2007).

Unfortunately, however, they do not provide details regarding agents expectations regarding the future. This problem is exacerbated in the current context, where assumptions regarding wage growth are likely to influence preferences toward indebtedness. And if one admits the importance of wage growth in the analysis, then other aspects of the cross-sectional policy environment begin to demand adjustment, such as the specification of tax rates and thresholds.

We have opted to calibrate the model against age profiles calculated from the cross-sectional micro data provided by the 2001/02, 2002/03, and 2003/04 waves of the Expenditure and Food Survey, adjusting all financial characteristics for wage growth of 1.9% per annum.[44] This figure for wage growth is on the high end of the historical variation, and reflects our current interest in considering whether, given plausible assumptions for individual expectations, contemporary consumer borrowing can be explained by the rational agent model described here.

Taxes and benefits in the model are also based upon an adjusted form of the UK transfer system as it was applied in 2003.[45] Regarding the adjustments made to the transfer system, comparisons between the simulated profiles and survey data for private and disposable income would be simplified if the wage growth indexing applied to the financial characteristics described by survey data was also applied to the various thresholds that are assumed for the tax system. This was not done, however, as such an assumption would provide a widely inaccurate reflection of the practical reality: threshold rates tend to grow at the rate of price inflation, and benefits by more than the rate of price inflation, but by less than wage growth. We have consequently held tax rates and thresholds fixed (in real terms), and projected welfare benefits to grow by 1 percent per year (less than the 1.9 percent assumed for wage growth).

The criteria that are assumed here for calibrating the model depend upon clear value judgements about agent expectations. Ideally, we would test how sensitive the analytical conclusions of this study are to the basis considered for calibration. That issue remains a subject for further research.

4.2 Calibration of the income process

The wage generating process was calibrated by comparing simulated and sample statistics for the distribution of household private non-property income by age. Comparisons based upon the distribution of household disposable non-property income then provide an indication of how closely the simulated tax and benefits policy reflects the practical reality, as described by survey data.

Four aspects of the wage generating process were subject to detailed calibration for each of singles

[44] Based on real household disposable (equivalised) income growth for the period 1971-2002, as reported in Figure 5.14 of Social Trends 34 (ONS publication). Growth calculated with respect to household age – i.e. $y_{t,i} = z_{t,i}.1.019^{(t-k(i))}$, where z denotes the respective financial characteristic (consumption, private income, or disposable income) as described by survey data, y denotes the value considered for calibration, i refers to the sample wave, $k(i)$ takes the value 20 for the 2003/04 EFS wave, 19 for the 2002/03 wave, and 18 for the 2001/02 wave, and t denotes household age.

[45] This approach may be contrasted with that of Nelissen (1998) who explores redistribution using a microsimulation model that is designed to capture the historical variation observed in Dutch tax and benefits policy.

and couples (and for each education subgroup). First, the parameters defining the initial distribution of wages $\left(\mu_{na,20}, \sigma^2_{na,20}\right)$ were selected to reflect statistics of the distribution for private non-property income at age 20. Second, the intertemporal persistence term $\left(\beta_{na_{i,t}}\right)$ was increased to motivate higher labour supply by the young, and to smooth the rate of labour market departure at older ages. Third, age specific dummy variables (one for each year) and the variance of the household specific disturbance term $\left(\sigma^2_{\omega,na_{i,t}}\right)$ were adjusted to match the age profile of the distribution of private non-property income to the profiles estimated from survey data. And finally, the (age independent) probability of a low wage offer (φ) was adjusted to match the level of the proportion of the respective population (singles or couples) employed – this parameter was introduced into the model as we were otherwise unable to capture the discretely lower rates of employment observed for single households (a limitation that is explained, in part, by the fact that we do not include an explicit allowance for child-care in our model).

4.3 Calibration of preference parameters

There are four preference parameters to calibrate, γ, ε, δ, and α. The parameter pair (γ, δ) tend to determine household preferences over feasible intertemporal expenditure paths, and the parameter pair (ε, α) affect preferences over feasible intratemporal consumption/leisure choices.[46] These parameters were adjusted to match the model against age profiles estimated for the full UK population.

For given values of the elasticity parameters γ and ε, we chose the discount rate, δ, to achieve the 'closest' match between the simulated and estimated age profiles for mean household consumption; and we chose the parameter α to match average retirement rates. Effectively this process defines the parameters (δ, α) as a function of (γ, ε). Additional criteria were therefore required to select the parameters (γ, ε).

It can be shown (see Sefton et al. (2006)) that increasing ε, decreases the demand for leisure relative to consumption for high income households (equivalent to later retirement in the model), but has the opposite influence on low income households. Hence, ε was adjusted to match the cross-sectional timing of retirement. Increasing γ tends to exaggerate the influence of disparities between the interest rate (r) and the discount rate (δ) on the time profile of consumption. Furthermore, as noted by Heckman (1974), consumption will tend to track labour income under the life-cycle hypothesis if leisure and consumption are direct substitutes in utility. A smaller value of γ, *ceteris paribus*, implies greater substitutability between leisure and consumption, and hence more pronounced income tracking. We therefore adjusted γ to fit the distribution of consumption about the mandatory retirement age, when labour changes most substantially. See Sefton et al. (2006) for detailed statistics indicating the sensitivity of the simulations

[46] This division does not strictly hold because the experience effect on wages invalidates two-stage budgeting. Nevertheless, it does provide a reasonable approximation, particularly toward the age of retirement, when the experience effect on wages has a dampened influence on individual behaviour.

to alternative assumptions regarding γ and ε.

4.4 The fit between simulated and sample age profiles

We begin by discussing statistics associated with calibration of the model to sample statistics estimated for the full UK population, before moving on to discuss calibrations for the education specific subgroups.

The representative sample

Following an extensive search, our preferred parameter values are reported in Table 4. The table is divided into three panels. The top panel reports preference parameters and other exogenously assumed population characteristics, the middle panel reports parameters for the wage generating process, and the bottom panel provides age specific dummy variables.

The calibrated parameter combination implies that consumption and leisure are direct substitutes, and is associated with an average intertemporal elasticity of substitution for consumption of 0.52, which lies within the range of values considered by the literature.[47] The parameters assumed for the wage generating process imply strong intertemporal persistence. The survival probabilities assumed for the analysis are based upon the probability that at least one member of a couple survives from age 20, and were calculated from statistics reported in the life tables published by the Government Actuary's Department for the UK for years 2001-2003, (the most recent period which GAD projections were available).[48]

The relation between the simulated statistics obtained from the model using the calibrated parameter values reported in Table 4, and the statistics calculated from survey data are displayed in Figures 8 to 11. The statistics calculated from the simulated data are referred to as "simulated statistics", and those calculated from survey data are referred to as "sample statistics". All monetary values are reported relative to average gross annual income for all full-time employees in the UK during 2003/04, equal to £22,724[49].

Figures 8 to 11 indicate that, in spite of the strong assumptions upon which the model is based, a close match was obtained between the simulated and sample statistics. Starting with Figure 8, the top panel indicates that a reasonably close match was obtained to the age profile of employment for both

[47] Calculated at population averages at age 45 for consumption (£557 per week), leisure (0.9), and the equivalence scale (1.804 * 470). Simulations undertaken by Auerbach & Kotlikoff (1987) are based upon an intertemporal elasticity of substitution of 0.25 (coefficient of risk aversion of 4), while Cooley & Prescott (1995) consider a value of 1. Grossman & Shiller (1981), Mankiw (1985) and Hall (1988) report econometric estimates for the intertemporal elasticity between 0 and 0.4, Blundell et al. (1994) report an estimate of 0.75, while Hansen & Singleton (1983), Mankiw et al. (1985), and Ziliak & Kniesner (2005) report estimates of approximately 1. Values of the coefficient of risk aversion required to explain the equity premium puzzle (Mehra & Prescott (1985)) imply very small elasticities of intertemporal substitution, although evidence from attitudinal surveys suggest that the value is unlikely to be less than 0.2 (Barsky et al. (1997)).

[48] On 31 January 2006, responsibility for the production of national life tables transferred to the Office for National Statistics (ONS). Current and historic life tables are still available on the GAD website at: http://www.gad.gov.uk/Life_Tables/Life_tables_background.htm.

[49] Average gross annual income for all full-time employees reported for winter 2003/04 (Labour Force Survey Historical Quarterly Supplement, Table 37, available from the National Statistics website: http://www.statistics.gov.uk/).

Table 4: Calibrated Model Parameters for Representative Sample of UK Population

General Model Parameters

intertemporal elasticity (gamma)	0.51	interest rate on investments (%)	4.00
intratemporal elasticity (epsilon)	0.55	interest rate on debt, lower bound (%)	7.00
utility price of leisure (alpha)	0.66	interest rate on debt, upper bound (%)	15.00
discount rate (delta)	0.98	wealth at age 20	0.00

Wage Generating Parameters

	singles	couples
mean log wage at age 20	5.00	5.50
standard dev of log wage at age 20	0.10	0.10
wage persistence (beta)	0.90	0.90
standard dev of log wage from age 21	0.35	0.30
probability of low wage offer (phi)	0.30	0.05

Age Specific Dummy Variables

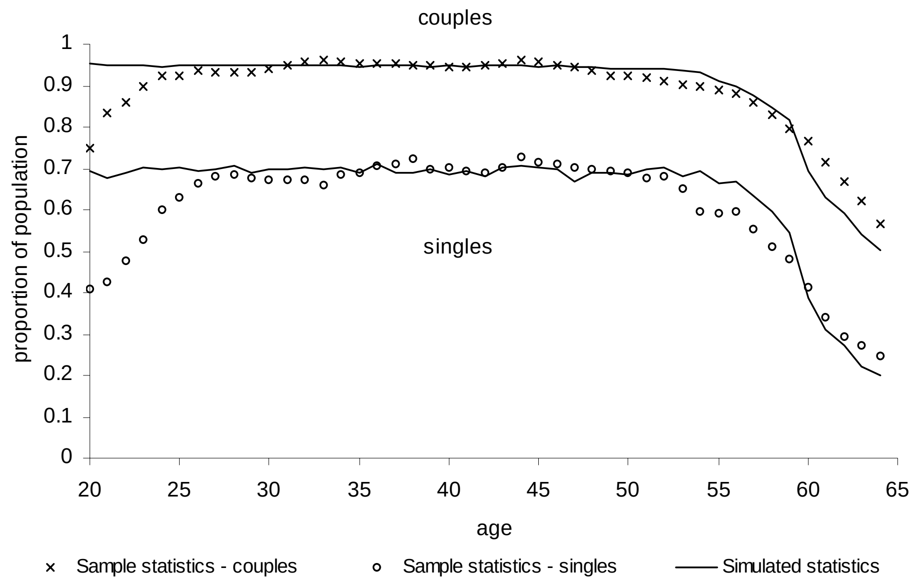
age	m(t) singles	m(t) couples	credit limit	grid points	age	survival rate*	grid points
20	1.000	6.942	19952.74	245586	65	0.997	45602
21	1.888	12.428	20158.45	245586	66	0.996	45602
22	3.872	18.968	20366.63	245586	67	0.996	45602
23	7.130	27.936	20577.35	245586	68	0.995	45602
24	11.554	39.991	20790.73	245586	69	0.994	45602
25	17.818	53.951	21006.85	982344	70	0.993	45602
26	25.833	71.015	21225.85	982344	71	0.991	45602
27	35.845	91.250	21447.86	982344	72	0.989	45602
28	47.371	114.223	21673.03	982344	73	0.987	45602
29	60.453	140.342	21901.54	982344	74	0.984	45602
30	74.685	169.305	22133.58	982344	75	0.981	45602
31	91.613	201.059	22369.38	982344	76	0.978	45602
32	109.382	236.094	22609.18	982344	77	0.974	45602
33	128.904	273.995	22853.27	982344	78	0.970	45602
34	148.544	314.290	23101.99	3929376	79	0.965	45602
35	169.814	356.355	23355.70	3929376	80	0.959	45602
36	190.578	399.815	23614.83	3929376	81	0.953	45602
37	211.919	442.538	23879.87	3929376	82	0.946	45602
38	232.691	487.035	24151.37	3929376	83	0.937	45602
39	255.758	531.893	24429.97	3929376	84	0.928	45602
40	279.013	577.142	24716.41	3929376	85	0.914	45602
41	302.059	622.234	25011.51	3929376	86	0.903	45602
42	325.779	664.874	25316.22	3929376	87	0.890	45602
43	352.518	704.218	25631.66	982344	88	0.876	45602
44	377.770	745.003	25959.07	982344	89	0.859	45602
45	400.999	782.421	26299.90	982344	90	0.846	22801
46	423.420	817.691	26655.81	982344	91	0.830	22801
47	442.641	850.879	27028.69	982344	92	0.810	22801
48	461.247	882.637	27420.72	982344	93	0.787	22801
49	477.074	909.949	27834.42	982344	94	0.772	22801
50	490.054	933.874	28272.66	982344	95	0.749	22801
51	498.650	949.910	28738.75	982344	96	0.731	22801
52	499.827	955.071	29236.48	245586	97	0.710	22801
53	492.924	949.269	29770.22	245586	98	0.692	22801
54	480.399	936.375	30344.99	245586	99	0.676	22801
55	458.006	910.519	30966.54	245586	100	0.650	22801
56	428.937	876.902	30437.11	245586	101	0.611	22801
57	385.408	822.972	29775.99	245586	102	0.578	22801
58	340.386	759.827	28962.93	245586	103	0.526	22801
59	296.949	697.309	27974.59	245586	104	0.475	22801
60	256.692	636.859	26784.15	245586	105	0.416	22801
61	219.200	578.145	22949.09	245586	106	0.349	22801
62	198.645	541.325	18459.73	245586	107	0.270	22801
63	176.663	502.568	13217.13	245586	108	0.187	22801
64	165.885	480.725	7107.51	245586	109	0.100	22801
					110	0.000	22801

m(t) = age specific dummy veriables in wage generating process

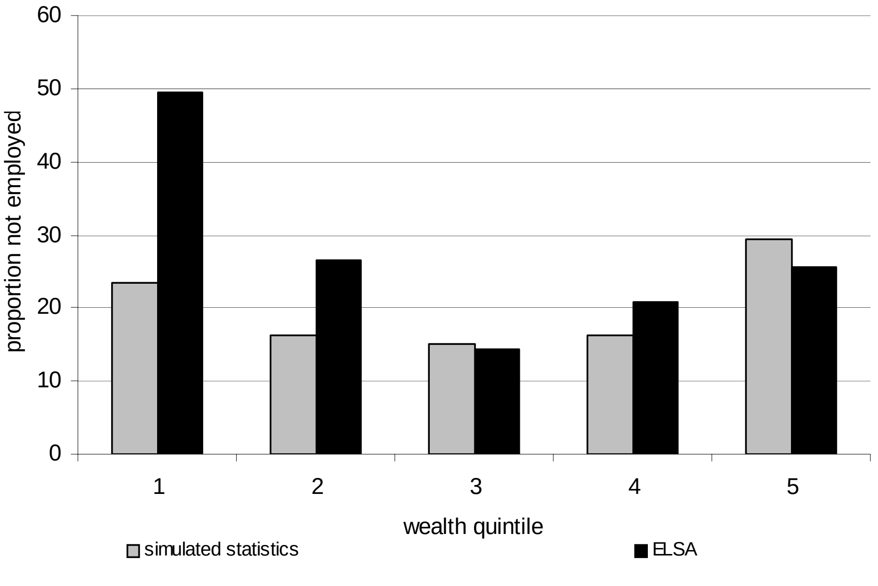
credit limit = maximum debt households can draw upon

grid points = no of points by age at which a solution is obtained to the utility maximisation problem (no allowance for personal pensions)

* survival rate = probability of surviving to age t+1 given survival to age t



Panel A: Proportion employed by age and relationship status



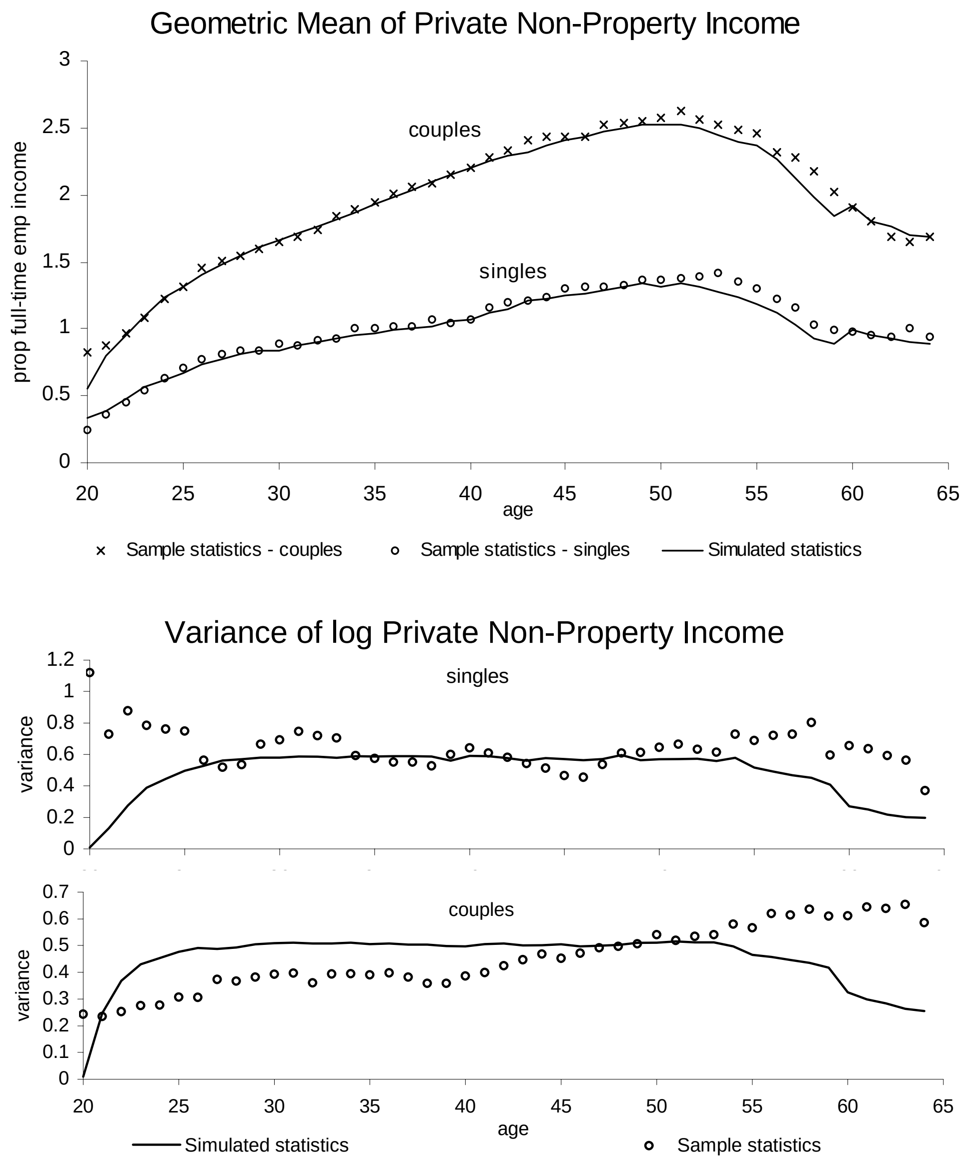
Panel B: Distribution of early retirement – ages 55 to 59

Notes: Sample statistics – age profiles calculated from Expenditure and Food Survey data, 2001/02, 2002/03, and 2003/04

Simulated statistics – age profiles generated from model, using calibrated parameters reported in Table 4

ELSA – percentage of men aged 55-59 economically inactive by wealth quintile, 2002/03 wave of the English Longitudinal Study of Aging, Table 4A.12, Marmot et al. (2003)

Figure 8: Timing of Retirement – simulated versus survey data

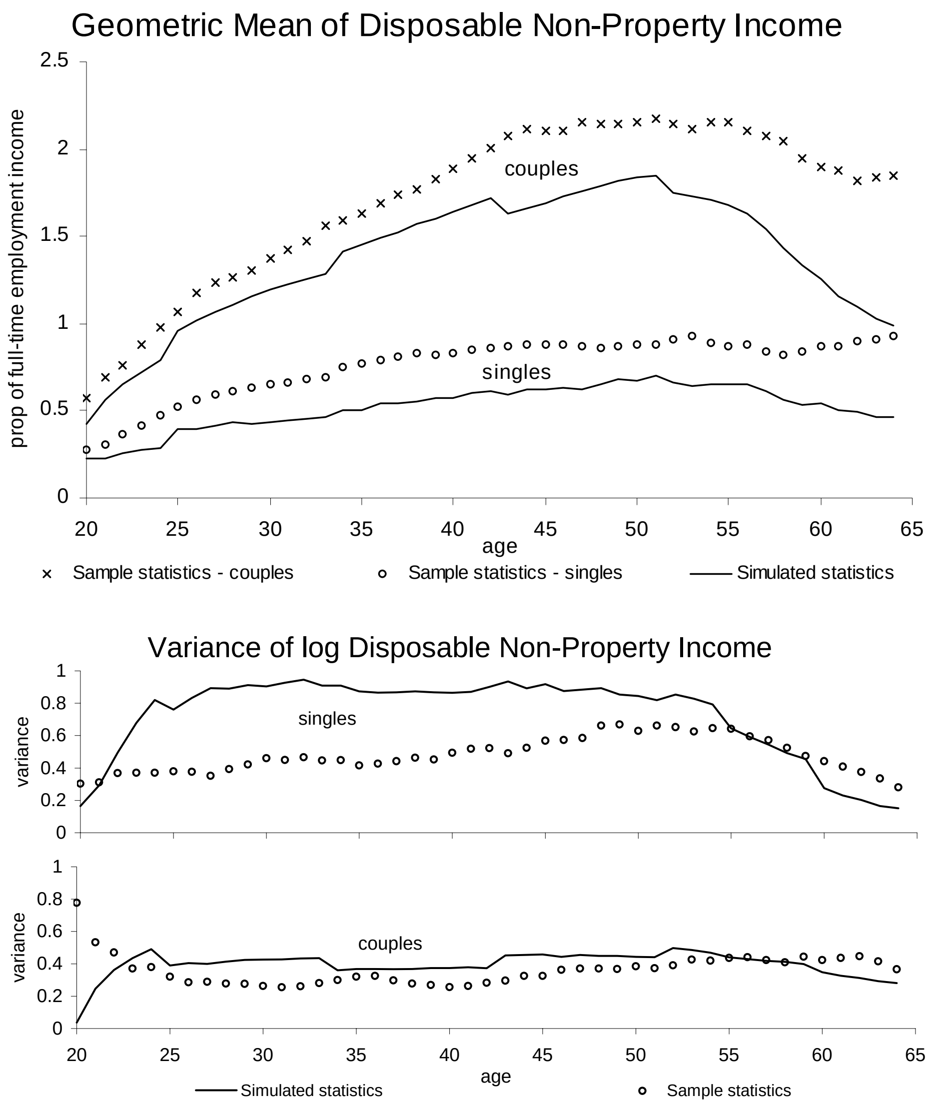


Notes: Sample statistics – age profiles calculated from Expenditure and Food Survey data, 2003/04

Simulated statistics – age profiles generated from model, using calibrated parameters reported in Table 4

Monetary values reported as proportions of average annual full-time employment income for 2003/04, equal to £22,724

Figure 9: Private Non-Property Income Profiles by Age – simulated versus survey data

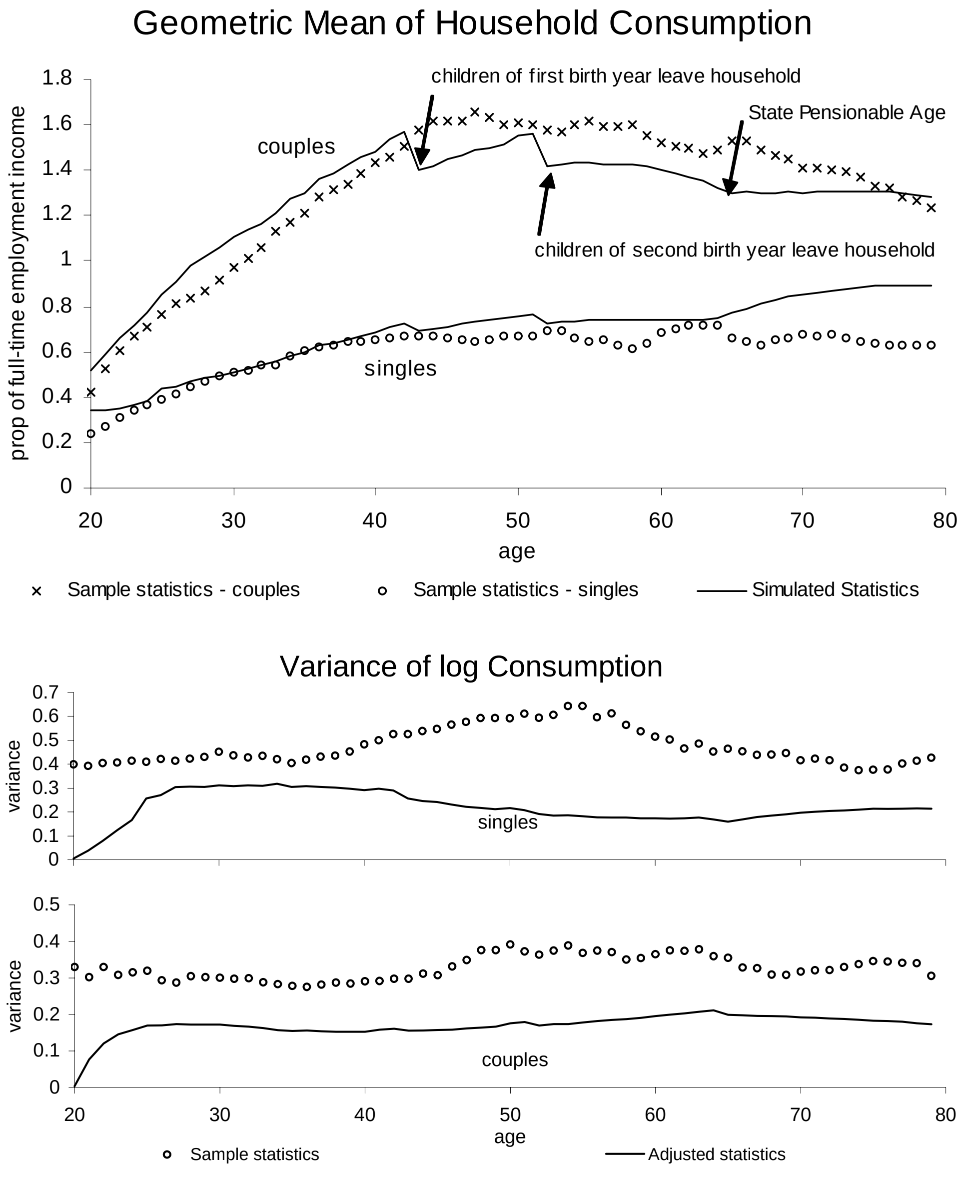


Notes: Sample statistics – age profiles calculated from Expenditure and Food Survey data, 2003/04

Simulated statistics – age profiles generated from model, using calibrated parameters reported in Table 4

Monetary values reported as proportions of average annual full-time employment income for 2003/04, equal to £22,724

Figure 10: Disposable Non-Property Income Profiles by Age – simulated versus survey data



Notes: Sample statistics – age profiles calculated from Expenditure and Food Survey data, 2003/04

Simulated statistics – age profiles generated from model, using calibrated parameters reported in Table 4

Monetary values reported as proportions of average annual full-time employment income in 2003/04, equal to £22,724

Figure 11: Consumption Profiles by Age – simulated versus survey data

singles and couples. The most pronounced departure between the simulated and sample statistics for the proportions of the population employed occurs between ages 20 and 25, with the simulation model generating excessive proportions of the population employed for both singles and couples. This is attributable to our omission from the model of education, and the focus of the model on labour supply, subject to an age independent probability of a low wage offer. The excess employment generated during early ages is likely to reduce the extent of indebtedness generated by the rational agent model, and should be borne in mind when interpreting the analysis presented in Section 7. In contrast, the simulation model appears to do a very good job of capturing the relation between age and retirement described by the sample statistics.

The bottom panel of Figure 8 indicates that the simulation model approximates the distribution of early retirement described by sample statistics, with earlier retirement at the distributional extremes, and later retirement in the middle quintiles. It is of note, however, that the model generates earlier retirement amongst households in the highest wealth quintile than it does for households in the poorest quintile, in contrast with the associated sample statistics. As noted in Section 4.3, this match is controlled by adjusting the intra-temporal elasticity of substitution between consumption and leisure – increasing ε tends to bring forward the retirement age of poorer households and delay the retirement of richer households. Our calibration sets $\varepsilon = 0.55$, a value that is at the upper end of the range identified in the related empirical literature (see, for example, Stern (1976)), and was selected after extensive experimentation.

With regard to the alternatives that were considered, we found that the match between the simulated and sample statistics reported in the bottom panel of Figure 8 could be improved by increasing ε. However, it was found that an 'unreasonably' high $\varepsilon = 0.9$ was required to approximate the distribution of early retirement as described by the sample statistics reported in the bottom panel of Figure 8. Furthermore, increasing ε had an important influence on other aspects of the model calibration – most notably the substitutability between consumption and leisure. Increasing ε, *ceteris paribus*, increases the substitutability between consumption and leisure. This distorts the consumption profile, an effect that can be offset by a coincident increase in γ. γ in turn influences the intertemporal elasticity of consumption, and the responsiveness of households to interest rates, which necessitates an adjustment of the discount factor δ to control for associated distortions to the consumption profile. Unfortunately, each of the adjustments referred to above are imperfect controls, in the sense that they also result in secondary effects. As a consequence, we found that increasing ε – while it could improve the distribution of early retirement as described in Figure 8 – obtained a poorer fit of the model in general.

The top panel of Figure 9 reports statistics for private (pre tax and benefit) non-property income. The two panels of the figure indicate that a close match was obtained to the age profiles of both the geometric mean and variance of private non-property income, for both singles and couples. In contrast, the simulated statistics for disposable (post tax and benefit) non-property income reported in Figure 10 suggest that the model understates the geometric mean described by survey data for both singles and couples, and overstates the associated variances. These disparities between the match obtained to private non-property income and disposable non-property income are to be anticipated, given the different adjustments applied to the financial characteristics described by survey data on the one hand, and to taxes and benefits on the other. Furthermore, they reflect the imperfect fit obtained between the simulated and sample statistics for household labour supply, the stylised specification assumed for taxation in the model, and the fact that the model makes no allowance for mis-representation, miscalculation, or reporting errors that are likely to affect survey data.

The top panel of Figure 11 indicates that the model captures the more pronounced hump shape by age observed for two adult households than for singles, which is in part attributable to the higher propensity of couples to have dependent children.[50] It was surprising to note that there is no appreciable fall in consumption about age 65 described by the sample statistics, an effect that is found in cross-sectional surveys for earlier time periods and has motivated substantial interest in the literature. In the current context, this observation made it necessary to set ε and γ to similar values, toning down the substitutability that is necessary to match similar models to earlier empirical data (see, for example, Sefton et al. (2007)). The profiles for the geometric mean of consumption suggest that the model generates a little too much saving by singles, and too little saving by couples. It is difficult to address these disparities in the current context, where the only differences between the two types of household are considered to be in terms of the wage generating process, tax treatment, and consumption needs. With regard to the statistics reported in the bottom panel of Figure 11, the model understates the variance of consumption by age for both couples and singles described by the survey data. This may indicate too much intertemporal smoothing in the simulation model, or too much income mobility. With regard to the second of these two possibilities, it is of note that we assume a high value for the intertemporal persistence of the wage generating processes, β, for both singles and couples.

As noted above, although the wealth data available for UK households were considered to be inadequate for the purpose of calibrating the model, imputed wealth data taken from the 2000/01 wave of the BHPS were considered following the model calibration to check whether the implications for saving could be considered sensible. The relevant statistics are reported in Figure 12. These suggest that the simulation model does a reasonably good job of capturing both the rate of accrual and dispersal of

[50] The sample statistics for geometric means of consumption by age for both singles and couples were proportionally adjusted to match the implied lifetime averages (weighting each age equally) to the associated simulated statistics. This adjustment, equal to a 21 percent increase for singles and a 3 percent decrease for couples, was applied to ensure that the calibration with respect to consumption was not affected by disparities associated with disposable income or the lifetime budget constraint that is applied in the model.

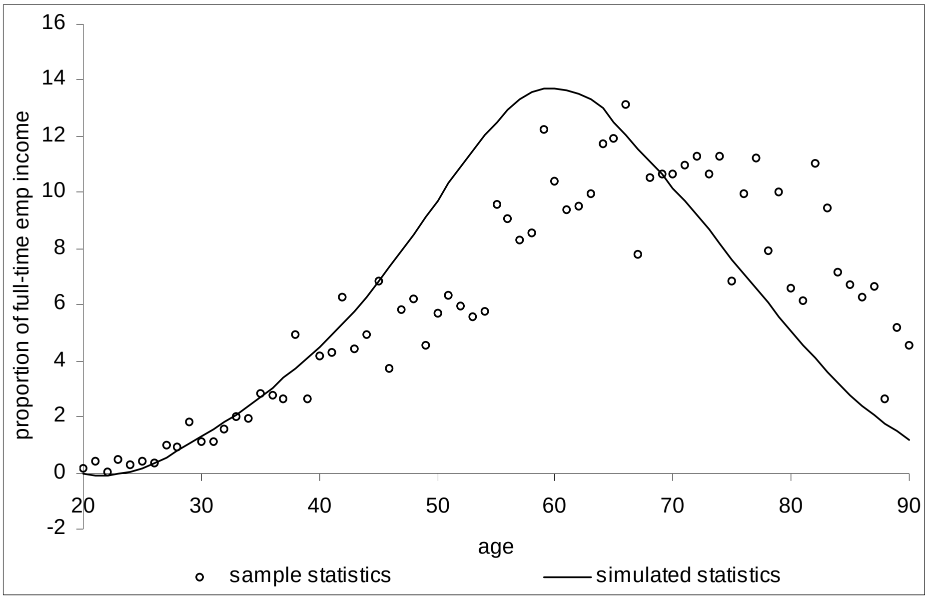


Figure 12: Wealth Profiles by Age – simulated versus survey data

wealth described by the associated survey data.

Calibrations distinguished by level of education

The parameters of the model that describe the wage generating process were adjusted to reflect sample statistics for private non-property household income of population subgroups distinguished by level of education. The education specific sample statistics were based upon the same EFS data as were considered for the full population, but divided into two subgroups distinguishing households by whether at least one member had full-time education beyond the age of 16. This divided the sample population into two roughly equal halves (44% of households being identified in the higher educated subgroup).

The education specific model parameters are reported in Table 5 and the relation between simulated and survey data are presented in Appendix C.

4.5 Summary

- We have calibrated the model described in Section 3 to survey data derived from the 2001/02, 2002/03, and 2003/04 waves of the Expenditure and Food Survey, adjusting the cross-sectional survey data to reflect wage growth of 1.9% per annum.
- The model was calibrated against age profiles that are representative of the full UK population, and for population subgroups distinguished by level of education.
- The model does a good job of capturing the moments against which it has been calibrated, despite the strong assumptions upon which it is based.

Table 5: Calibrated Model Parameters for Population Sub-samples Distinguished by Level of Education

General Parameters	full population		lower educated		higher educated	
	singles	couples	singles	couples	singles	couples
mean log wage at age 20	5.00	5.50	4.80	5.65	5.00	5.50
sd log wage at age 20	0.10	0.10	0.10	0.10	0.10	0.10
wage persistence (beta)	0.90	0.90	0.90	0.90	0.90	0.90
sd log wage from age 21	0.35	0.30	0.35	0.27	0.35	0.27
probability of low wage offer (phi)	0.30	0.05	0.39	0.09	0.20	0.05

Age Specific Dummy Variables	full population		lower educated		higher educated	
age	m(t) singles	m(t) couples	m(t) singles	m(t) couples	m(t) singles	m(t) couples
20	1.000	6.942	1.000	6.942	1.000	6.942
21	1.888	12.428	1.915	11.648	2.160	12.180
22	3.872	18.968	3.333	16.623	4.803	18.853
23	7.130	27.936	5.667	23.386	9.025	28.039
24	11.554	39.991	8.962	32.155	14.828	40.313
25	17.818	53.951	13.563	41.949	22.946	54.928
26	25.833	71.015	19.517	53.950	33.335	73.220
27	35.845	91.250	27.225	68.290	46.165	95.292
28	47.371	114.223	36.054	84.736	61.040	120.643
29	60.453	140.342	46.541	103.488	77.996	150.246
30	74.685	169.305	58.028	124.662	96.963	183.926
31	91.613	201.059	72.310	148.231	118.044	221.744
32	109.382	236.094	86.724	174.134	140.568	264.315
33	128.904	273.995	102.221	203.103	163.853	310.583
34	148.544	314.290	118.199	234.732	186.978	359.801
35	169.814	356.355	136.924	268.693	211.251	410.286
36	190.578	399.815	154.524	304.855	235.765	461.407
37	211.919	442.538	173.682	341.884	261.163	510.913
38	232.691	487.035	191.710	379.097	289.029	563.540
39	255.758	531.893	211.021	416.155	317.602	616.604
40	279.013	577.142	232.272	452.634	346.873	669.227
41	302.059	622.234	253.435	486.920	380.501	722.000
42	325.779	664.874	273.954	518.919	414.330	773.376
43	352.518	704.218	296.138	549.515	447.590	820.307
44	377.770	745.003	317.164	579.258	478.243	868.305
45	400.999	782.421	336.534	607.071	505.309	911.211
46	423.420	817.691	354.307	634.259	527.372	951.720
47	442.641	850.879	372.264	658.904	545.228	992.194
48	461.247	882.637	390.261	684.299	560.700	1031.858
49	477.074	909.949	407.684	705.680	575.516	1069.148
50	490.054	933.874	421.737	722.592	593.797	1107.661
51	498.650	949.910	431.431	735.898	606.456	1136.644
52	499.827	955.071	434.633	747.422	612.712	1155.095
53	492.924	949.269	437.349	750.009	609.939	1162.569
54	480.399	936.375	433.582	751.741	601.068	1157.979
55	458.006	910.519	425.169	748.887	570.513	1135.414
56	428.937	876.902	411.171	727.800	533.139	1100.669
57	385.408	822.972	386.459	702.726	497.002	1039.218
58	340.386	759.827	356.158	667.249	450.114	962.527
59	296.949	697.309	314.400	619.415	389.480	884.859
60	256.692	636.859	277.678	566.689	327.735	806.563
61	219.200	578.145	235.573	524.341	288.767	721.918
62	198.645	541.325	206.234	496.490	257.793	649.256
63	176.663	502.568	172.659	473.853	234.049	588.958
64	165.885	480.725	149.299	451.749	226.356	533.623

m(t) = age specific dummy veriables in wage generating process

5 Dis-aggregating Net Wealth to Identify Housing and Collateralised Debt

Owner occupied housing is a special type of asset for two reasons. First, it is "lumpy" in a way that few assets in the typical household balance sheet are. The indivisible nature of housing gives rise to a host of subsidiary complexities including minimum deposits, mortgage debt, and non-trivial transactions costs. Secondly, housing is desired for the services that it provides, in addition to the purely financial considerations that can reasonably be assumed to govern demand for other types of investment. This means that if housing is included in a model of rational behaviour, then it should enter as a term in the assumed utility function. These considerations complicate the inclusion of housing in a dynamic programming model of household savings behaviour, such as the one used for the current study.

Specifically, the current study explores the interactions between saving and unsecured indebtedness implied by a rational agent model of the household, and how these observations relate to associated survey data. In this context, housing is important because of its prominence in the aggregate stock of wealth held by households in the UK, and because of its bearing on secured debt. Unfortunately, models of the type considered for this study are numerically demanding, and are made more so by the inclusion of "lumpy" terms in the utility function. To avoid making the model unwieldy, we have had to make a choice between two options: to include the lumpiness of housing, but omit it from the utility function, or vice versa.

In our second report submitted under the current project to the DWP in June of 2005, we included housing in the utility function, but omitted its associated lumpiness. The model used to undertake the analysis took approximately two days to solve, and the omission of lumpiness from the housing asset was considered inherently problematic. We have subsequently agreed to include the lumpiness of housing, but to omit it from the rational decision making environment. In addition to facilitating consideration of secured debt, this amendment has enabled us to reformulate the model to better capture uncertainty in household demographics, which has an important bearing on the issue of over-indebtedness and household savings (as is discussed at length in the sections above).

We use a nested logit model comprised of two levels to project measures of housing wealth and secured debt on the basis of the other household characteristics that are generated by the dynamic programming model. The first level of the nested model identifies whether a household is an owner occupier and, if so, the second level identifies whether they hold mortgage debt. Linear regression models are then used to identify the value of a household's housing wealth, and the value of any mortgage that they may hold.

Although this analytical approach captures aspects of the housing market that were omitted from our second report (such as the lumpiness of housing, and an explicit consideration of mortgage debt), it is important to note that the approach adopted here to model housing represents an important methodological departure from the remainder of the study. The basic premise of the analysis is to consider the relation between debt and savings, as determined by rational economic theory, and how this bears upon over-indebtedness and welfare in retirement. Modelling housing using reduced form models is a clear departure from rational economic theory, which should be remembered when interpreting the results obtained.

The reduced form models used to project measures of housing wealth and collateralised debt were estimated econometrically using survey data from the BHPS, again adjusted for real wage growth of 1.9% per annum.[51] Regression statistics are reported in Tables 6 and 7.

Table 6 indicates that the logit regression considered for analysis does a very good job of capturing variation observed in the data. The regression statistics indicate that the probability of being a house owner initially increases with age before levelling out, and finally declining later in life. The probability of being a house owner also increases, but at a decreasing rate, with labour income and net assets. Singles, and those with low net liquid wealth are less likely to own their own home, an issue that is offset somewhat as the value of net debts rise. Finally, house ownership in the previous year is a strong indicator of house ownership in the current year.

The tobit regression statistics reported in Table 6 indicate that the various characteristics referred to above have a similar effect on the likely value of housing wealth as they do on the probability of owning a home. A conspicuous exception is the number of dependent children in a household, which was not found to have a significant influence on the probability of house ownership, but tends to increase the value of owner occupied housing. This is consistent with the increase in housing needs that comes with a larger family.

Table 7 indicates that, amongst house owners, the probability of holding a mortgage is high at young ages and declines fairly sharply after age 40. Those with higher incomes tend to also have a higher probability of holding a mortgage, and the probability of holding a mortgage in any year is strongly correlated with holding a mortgage in the preceding year. Finally, house owners with low net liquid wealth are also far more likely to hold a mortgage than otherwise similar individuals. Finally, the tobit statistics reported in Table 7 indicate that the value of a mortgage tends to decrease with the age of a household, and increase with labour income, house value, and low net liquid wealth.

[51] The sample was screened to omit those observations with missing data, resulting in a sample size of 6290 used for estimation.

Table 6: Regression Statistics for Equations Used to Simulate Housing

Logit model for house ownership						
	full population		lower educated		higher educated	
	estimate	s.e.	estimate	s.e.	estimate	s.e.
const	-18.49320	3.631E+00	-24.29840	6.516E+00	-16.98370	5.141E+00
age	1.51836	3.384E-01	1.99973	5.860E-01	1.40389	5.024E-01
age^2	-0.04164	1.099E-02	-0.05443	1.834E-02	-0.03952	1.727E-02
age^3	4.555E-04	1.481E-04	5.919E-04	2.387E-04	4.488E-04*	2.473E-04
age^4	-1.780E-06	7.064E-07	-2.287E-06	1.104E-06	-1.855E-06*	1.252E-06
lab inc	2.163E-05	6.345E-06	2.151E-05*	1.572E-05	2.176E-05	7.996E-06
lab inc^2	-8.587E-11	2.444E-11	-4.451E-11*	1.707E-10	-9.036E-11	2.674E-11
dum(single)	-0.88502	1.862E-01	-1.06605	3.034E-01	-0.79034	2.456E-01
net assets	1.894E-05	2.195E-06	1.836E-05	2.740E-06	2.324E-05	4.615E-06
net assets^2	-5.836E-12	6.997E-13	-7.043E-12	1.047E-12	-6.672E-12	2.067E-12
net debt	4.439E-05	1.424E-05	6.447E-05	3.209E-05	2.839E-05*	1.561E-05
dum(owner t-1)	4.87517	1.990E-01	5.86396	3.359E-01	3.95630	2.481E-01
dum(low wealth)	-4.01533	2.610E-01	-5.20386	4.591E-01	-2.98517	3.267E-01
correct predictions	0.9719		0.9782		0.9604	
observations	6290		3663		2627	
prop. with home	0.6458		0.5495		0.7800	

Tobit regression for house value						
	full population		lower educated		higher educated	
	estimate	s.e.	estimate	s.e.	estimate	s.e.
const	45245.50	1.186E+04	60548.30	18152.40	25455.90*	17808.50
age	-1163.88	4.752E+02	-1278.90	637.61	-302.64*	806.71
age^2	3.23396*	4.461E+00	2.26200*	5.56711	-4.35537*	8.19531
lab inc	1.33158	7.995E-02	0.21441*	0.19725	1.37654	0.11139
lab inc^2	-3.213E-06	4.752E-07	9.492E-06	2.384E-06	-3.522E-06	5.366E-07
children	16634.60	3.552E+03	20423.00	5140.40	16750.40	5164.17
children^2	-3396.12	1.130E+03	-4528.44	1552.44	-2768.53*	1732.33
net assets	8.590E-01	1.157E-02	9.027E-01	0.01659	7.932E-01	0.01758
net assets^2	-4.479E-08	7.004E-09	-6.209E-08	1.063E-08	-2.101E-08	9.513E-09
net debt	9.226E-01	1.045E-01	4.917E+00	3.731E-01	5.823E-01	1.139E-01
lab inc x net assets	-3.363E-07	1.225E-07	7.482E-07	3.344E-07	-2.392E-07*	1.460E-07
dum(low wealth)	-1.337E+05	4.143E+03	-1.526E+05	5.899E+03	-1.126E+05	6.139E+03
log likelihood	-52620.20		-25890.60		-26568.10	
Sigma**	86582.6	976.792	77485.7	1247.49	91166.2	1443

dum(.) denotes dummy variable

* estimate not statistically significant at 95% confidence interval

** estimated standard deviation of residuals

low wealth defined as net liquid wealth under £5,200 in value

net liquid wealth = securities + housing - unsec debt - mortgage

net assets / debt taken as + / - measures of net liquid wealth

Table 7: Regression Statistics for Equations Used to Simulate Mortgages

Logit model for mortgage						
	full population		lower educated		higher educated	
	estimate	s.e.	estimate	s.e.	estimate	s.e.
const	-13.11330	4.100E+00	-15.86430	6.733E+00	-13.76850	5.892E+00
age	1.07101	3.605E-01	1.25962	5.684E-01	1.17780	5.430E-01
age^2	-0.03277	1.136E-02	-0.03769	1.727E-02	-0.03715	1.794E-02
age^3	4.046E-04	1.522E-04	4.579E-04	2.239E-04	4.791E-04*	2.518E-04
age^4	-1.801E-06	7.334E-07	-2.001E-06*	1.048E-06	-2.252E-06*	1.270E-06
lab inc	2.218E-05	4.244E-06	1.309E-05*	9.415E-06	2.336E-05	5.582E-06
lab inc^2	-1.367E-10	3.229E-11	-2.909E-11*	1.133E-10	-1.478E-10	3.792E-11
dum(mortgage t-1)	2.95662	1.143E-01	3.28485	1.704E-01	2.66982	1.564E-01
dum(low wealth)	4.24672	7.402E-01	4.30573	1.083E+00	4.16113	1.023E+00
correct predictions	0.87272		0.88028		0.86920	
observations	4062		2013		2049	
prop. with a mortgage	0.52120		0.37904		0.54219	

Tobit regression for mortgage value						
	full population		lower educated		higher educated	
	estimate	s.e.	estimate	s.e.	estimate	s.e.
const	-19173.70*	14435.50	2099.76*	24483.20	-86413.10	19662.30
age	2969.73	647.64	1068.56*	1023.32	6957.92	942.70
age^2	-61.85930	6.71565	-37.95300	9.96360	-111.50200	10.53000
lab inc	7.462E-01	4.909E-02	1.115E+00	1.046E-01	5.514E-01	5.651E-02
house value	0.07607	0.00722	0.05071	0.01254	0.10001	0.00893
dum(low wealth)	3.224E+04	4.959E+03	4.783E+04	8.954E+03	2.503E+04	5.796E+03
log likelihood	-27423.20		-10057.00		-17326.10	
Sigma**	69498.6	1126.87	74200.5	2031.92	65374.3	1314.17

dum(.) denotes dummy variable

* estimate not statistically significant at 95% confidence interval

** estimated standard deviation of residuals

population restricted to home owners

low wealth defined as net liquid wealth under £5,200 in value

net liquid wealth = securities + housing - unsec debt - mortgage

Source: Author calculations using data from waves 1 to 10 of the BHPS

5.1 Summary

- Housing is of particular interest in any analysis of private sector indebtedness, due to its indivisible nature, and the associated implications for mortgage debt.
- The indivisible nature of housing complicates its inclusion in a rational agent model, as does the consumption by individuals of housing services.
- An adequate treatment of housing within the rational agent framework was consequently considered to lie beyond the scope of the current study.
- The incidence of housing and mortgage debt is modelled in the current project by reduced form regression equations, that were estimated from BHPS survey data.
- The regression coefficients presented here are consistent with *a priori* expectations.

6 Intuition Behind the Life-cycle Model of Behaviour

This section gives a brief review of the intuition that lies behind the findings that are reported in Sections 7 and 8. A reader who is familiar with utility theory and the life-cycle model may omit this section without handicap.

The model used to undertake the analysis reported in the current study (described in Sections 3 and 4) implies that, all else being equal, an individual would prefer to smooth their consumption between two periods, rather than to consume a large amount in one period and very little in another. This is a property of the function that is assumed to describe individual preferences, and the parameters that were found to best reflect the survey data against which the model is calibrated. In this context, an individual will choose to spend more than their income in any given period if they expect that their consumption "needs" will decline, and/or their income will be higher in future periods.

Consumption needs are considered to be defined by the numbers of adults and children in a household, by labour supply during the working lifetime, and by the time of death. Three of these four factors are subject to uncertainty, and the fourth (labour supply) is a choice variable. Expectations regarding the timing of death influence saving incentives for retirement – the longer an individual believes that they are going to live, the greater the incentive to save, which competes with immediate consumption for available credit. In contrast, labour supply tends to increase demand for immediate consumption, as the preferences considered for analysis imply that consumption and leisure are direct substitutions. This may arise, for example, due to the costs of working or the desire to unwind after work through the pursuit of expensive leisure activities. Of course, labour supply also results in higher disposable income, so that the aggregate effect of working on wealth is usually positive.

The impact of the numbers of adults and children in a household on saving and indebtedness are more complex. An increase in household size raises consumption needs through the assumption of the McClement's equivalence scale, and also raises disposable income through the demographic dependance assumed for the tax and benefits system. With regard to the number of dependent children, the adjustments assumed for the tax system more than off-set changes to the assumed equivalence scale, so that households with dependent children tend to be better off than those without. With regard to the number of adults, the disposable income of couples is augmented relative to singles by both the tax system and the wage generating process, which imply that couples tend to be better off than otherwise similar singles.

These assumptions drive the incidence of indebtedness implied by the rational agent model. In the case of young single adults, for example, the anticipation that they will marry and obtain a dependent child at some time in the future gives rise to the expectation of an improvement in their financial circumstances. As noted above, the assumed preference structure implies that these adults will try to smooth their consumption intertemporally, going into debt when young with the expectation of paying the debt off when their circumstances improve. Alternatively, a household that is comprised of an adult couple when young may choose to go into debt to obtain an immediate benefit from the income growth that is embodied by the assumed wage generating process. If this household subsequently suffers a marital dissolution, then their indebtedness may be exaggerated, as described above for young single adults. Furthermore, households that experience a low wage offer will also expect their incomes to rise in the future, which motivates dis-saving in the model.

Although the factors referred to above are easily described, it is not possible to come to any conclusions regarding the magnitude of the respective effects without detailed simulations. That is the subject of discussion in the next section.

7 Saving and Indebtedness in a Rational Agent Model

This section focuses upon describing the behaviour toward saving and indebtedness implied by our rational agent model, and relating the behaviour toward indebtedness to contemporary survey data. The section begins by describing the behaviour implied by the rational agent model calibrated to the full UK population. We then focus upon the influence of education status on saving and borrowing, before exploring the related implications for housing and secured debt. A brief summary is included at the end of the section. Implications of the model for the related issues of over-indebtedness and poverty are explored in Section 8.

7.1 Behaviour a representative population cohort

Following the discussion of survey data that is reported in Section 2, the current discussion focusses primarily upon indebtedness defined in terms of net liquid wealth, which differs from total household wealth in that it omits accrued rights to private pensions. Unless otherwise stated, any reference to 'indebtedness' that is made below should be interpreted as a reference to negative net liquid wealth.

The rational agent model calibrated to the full UK population that is described in Sections 3 and 4 was used to generate life histories for a cohort of 20,000 households. Analysis of these life histories revealed that 100% of simulated households chose to be net indebted at some point in their lives. The arithmetic mean maximum debt during the simulated lifetime of the full population is £9,560, and the median debt held at any age of the simulated lifetime is £4,490, which are respectively approximately one half and one quarter of the age specific credit limits that are permitted under the model.

The current subsection begins by clarifying the definitions of the characteristics that are generated by the simulation model, before placing the behaviour generated by the model in context of related survey data. This discussion helps to identify some of the important limitations of the simulation model, relative to the practical reality. The subsection then undertakes a detailed analysis of the various motives that underly the behaviour toward savings and indebtedness that is implied by the rational agent model calibrated to the full UK population.

The definition of measures of wealth generated by the rational agent model

As discussed in Section 4.3, the rational agent model considered here was calibrated to reflect age profiles for means and variances of employment, income and consumption calculated from cross-sectional survey data. The financial data that were derived from the cross-sectional surveys were age adjusted for wage growth of 1.9% per annum to reflect the view that expectations regarding wage growth are likely to influence attitudes toward indebtedness in practice. To facilitate comparisons with contemporary survey data for financial stocks and flows, we have discounted the simulated statistics that are reported here to off-set the assumed wage growth.

Furthermore, comparisons between the extent of indebtedness generated by our rational agent model, and observations drawn from survey data are complicated by the nature of the composite net liquid wealth variable that is endogenous to the model. The *net liquid wealth* variable considered by the model represents the net non-pension worth of households, equal to housing equity less mortgage debt plus financial assets (e.g. shares and bank deposits) less unsecured loans. The model assumes that the same rate of interest accrues to housing equity, mortgages and financial assets, so that the household balance sheet is affected by the aggregate of these three – referred to below as *net liquid assets* – and not the allocation between them. Furthermore, it is assumed that mortgages can be freely adjusted but cannot exceed the value of housing equity, that other financial assets are completely liquid, and that a higher rate of interest is charged on *unsecured debt* than on net liquid assets. No account is taken of interest free periods for loan repayment, transactions costs involved in the management of cash-flows over the short term, imperfect substitution between alternative methods of payment, or any other motive to hold both net liquid assets and unsecured debts at the same time. Given these assumptions, there is no need to take separate account of net liquid assets and unsecured debt, as a rational household will necessarily choose to consume out of net liquid assets before resorting to an unsecured loan.

The population cohort that was generated by the model can consequently be divided by age into two subgroups, distinguishing households by whether they hold positive net liquid wealth. Those with positive net liquid wealth are considered to hold no unsecured debt, so that their net liquid assets are equal to their net liquid wealth. Similarly, those with negative net liquid wealth are considered to hold no net liquid assets, so that their unsecured debts are equal to their (negative) net liquid wealth. The following discussion attempts, as far as the data permit, to obtain definitions of liquid assets and unsecured debt that match those assumed for the simulation model.

Comparisons with the National Accounts

To compare the model against contemporary statistics reported by the National Accounts, age specific population averages were weighted to match the age profile described by the 2001 Census (the most recent Census available at the time of writing). The National Accounts data for 2001 indicate that aggregate disposable non-profit income was £563 billion, unsecured debt was £159 billion, and net liquid assets were £3,019 billion.[52] In contrast, the data generated by the model give an aggregate for disposable non-profit income of £356 billion, unsecured debt of £16 billion, and £1,209 billion for net liquid assets. All three population aggregates generated by the rational agent model are consequently less than the associated statistics derived from the National Accounts, with the widest proportional margin observed for unsecured indebtedness.

In Section 4.4, it is shown that the distribution of disposable income generated by the model describes an age profile for the geometric mean that is below that described by the survey data against which the model was calibrated for the entire working lifetime. This is attributed to the different adjustments applied to the financial characteristics described by survey data on the one hand, and to taxes and benefits on the other. Bearing this in mind, it is of little surprise that all three population aggregates that are reported above for the simulated cohort should understate the respective National Accounts statistics.

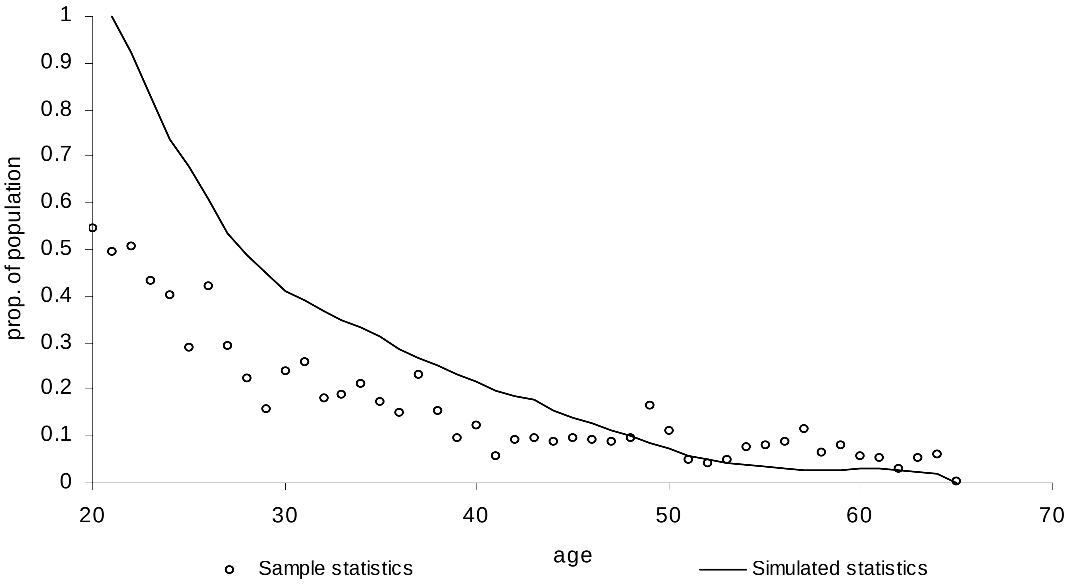
If the scale of disposable non-property income were solely responsible for the disparities observed in the aggregates for net liquid assets and unsecured debt, then we should expect that all three population aggregates generated by the model would differ in similar proportion to the National Accounts. This, however, is not the case. The population aggregates reported above indicate that the model understates the National Accounts data by 37% for disposable non-property income, by 60% for net liquid assets, and by 90% for unsecured debt. Hence, after adjusting by the scale of disposable non-property income, aggregate net liquid assets generated by the model are 63% $((1 - 0.6)/(1 - 0.37))$ of the National Accounts total, and unsecured debts are just 16% of the National Accounts total.

These statistics consequently suggest that the simulation model understates both the value of assets and the value of debt relative to disposable income described by the National Accounts. Before discussing the various possible explanations for, and implications of these observations, it is useful to consider the relation between data generated by the model and micro data reported by the BHPS.

Comparisons with BHPS data

In both our rational agent model and the BHPS survey data, most unsecured debt is taken on at young ages, when wages are relatively low, and consumption needs (as represented by the number of dependent children) are relatively high. The age specific incidence of indebtedness simulated by the

[52] ONS codes considered for analysis: Unsecured debt = NNRG + NNRK + NNRU; Liquid assets = NNML (total financial assets) - NPYL (insurance reserves) + CGRP (non-financial tangible assets, largely comprised of housing) - (NNQC + NNSQ + NNRQ + NNRR + NNRS) (long term debt and loans secured against property). Disposable non-property income = RPQK (total disposable income) - ROYL (property income received) + ROYT (property income paid) - NRJN (operating surplus) + ROYH (mixed income).



Notes: Sample statistics – author calculations using BHPS survey data

Simulated statistics – age profiles generated from model, using calibrated parameters reported in Table 4

Net debtors identified as households with negative net liquid wealth

Figure 13: Proportion of Population Net Debtors by Age – simulated and sample data

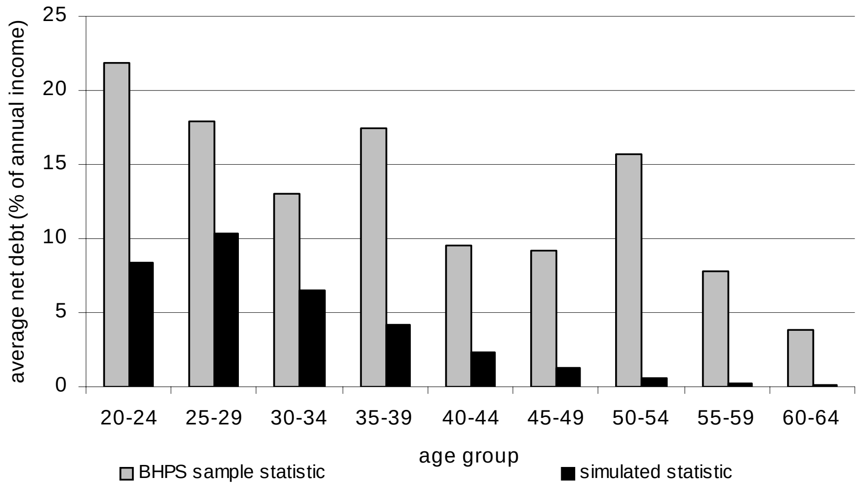
model is reported in Figure 13, alongside associated statistics calculated from BHPS data.

Figure 13 indicates that both the simulation model and the BHPS statistics suggest a smooth reduction in the proportions of the population indebted by age, with the proportion starting out at 100% in the simulations, and at 50% in the survey data. Hence, the proportion of the population indebted by age is higher in the simulation model for much of the life-course reported in the figure. From age 45, both the simulated and sample statistics indicate that the proportion of the population indebted is 10% or less.

The simulation model could both over-state the incidence of indebtedness (as indicated in Figure 13) and understate the aggregate value of debt in the private sector (as noted above with reference to National Accounts data), if the average value of debt per debtor was smaller in the simulated cohort than observed in practice. To test this proposition, Figure 14 plots the average debt per debtor for five year age bands calculated from the simulated and survey data.

Figure 14 reveals that the average net debt per debtor by age is lower in the simulation model than in the BHPS data. Simulated indebtedness rises from ages 20-24 to ages 25-29, and then declines toward retirement. This is in contrast to the survey data, which show the most substantial net debts at the youngest ages, and an erratic downward trend with age.

It is difficult to determine by comparing the statistics reported in Figures 13 and 14 whether the simulation model implies a smaller aggregate for unsecured debt than described by the BHPS data.



Sample statistics:	author calculations using BHPS survey data
	net assets calculated as aggregate of securities plus housing wealth less unsecured debt and mortgages
	expressed as percentages of average annual gross household income for ages 20-64, equal to £23,208
Simulated statistics:	author calculations using simulated data, based on calibrated parameters reported in Table 4
	age specific statistics discounted for 1.9% annual wage growth
	expressed as percentages of average annual gross household income for ages 20-64, equal to £30,487

Figure 14: Average Net Debt per Debtor (liquid wealth) – simulated and sample statistics

To facilitate comparison with the aggregates reported in relation to the National Accounts data, the age specific averages for unsecured debt calculated from BHPS data were weighted to reflect the 2001 Census data. This calculation obtained an aggregate value for unsecured debt of £8 billion, which is approximately one half of the aggregate calculated from data generated by the rational agent model (as discussed above). It is of note that this is in direct contrast to the comparisons with National Accounts data, which overstated by a wide margin the aggregate for unsecured debt implied by the the rational agent model.

Reconciling macro and micro statistics

The observations reported above suggest that the simulation model under-states the extent of household indebtedness reported in National Accounts data, and at the same time over-states the extent of indebtedness described by BHPS survey data. One of the reasons why the measures of aggregate unsecured debt reported from the BHPS and the rational agent model understate the measure reported from the National Accounts is that, although net liquid wealth per household can be identified for both the simulated cohort and the BHPS micro data, the same is not true for the National Accounts. The measure of aggregate unsecured debt reported from the National Accounts is consequently the total of all unsecured debt, regardless of any off-setting liquid assets that some households hold. Although this issue would be of little practical importance if the assumptions upon which the rational agent model is based were valid, micro data suggest that households in practice do tend to hold both net liquid assets and unsecured debts at the same time.

An appreciation for the scale of this issue can be derived from BHPS data, which provide the information necessary to calculate population aggregates for unsecured debt both net and gross of coincident holdings of liquid assets. The gross measure of unsecured debt calculated from the BHPS data (after weighting the population to reflect the age structure to match the 2001 Census), is £35 billion, and although this continues to substantially under-state the associated National Accounts figure, it is more than 4 times the population aggregate calculated for net unsecured debt (as reported above). Indeed, if the same factor as implied by the BHPS data is used to re-scale the measure of aggregate unsecured debt calculated for the simulated data, and if we also adjust for the understatement of disposable income by the simulation model, then we obtain £111 billion as an approximate for gross indebtedness, which accounts for 70% of the aggregate reported in the National Accounts.

The disparity identified above between the extent of aggregate indebtedness implied by macro and micro data is well-recognised in the literature, and is commonly attributed to deficiencies in the micro data. This is because financial institutions are consulted to obtain data regarding personal sector indebtedness for use in the National Accounts, whereas micro data sources rely upon the responses of individuals to survey questionnaires. It is of note in this context that, although details regarding the value of mortgage debts are commonly found to match fairly closely between the two data sources, micro data usually under-represent the value of unsecured indebtedness described by the National Accounts. This has been taken to suggest that individuals may be unable or unwilling to accurately report the value of various forms of unsecured loans that they hold (see Antoniewicz et al. (2005) for details regard data in Canada, Italy, and the United States).

If the micro data are to be considered unreliable, then what can we infer from the relation between simulated and National Accounts aggregates that are reported above? On the one hand, if we take the aggregates reported at face value, then it would appear that households in the rational agent model may engage in less income smoothing than is the case in practice (as they make less use of unsecured debt). The statistics for the model calibrations that are reported in Section 4.4 provide some support for this proposition (see Figures 10 and 11). Nevertheless, the differences between the simulation model and the data against which it was calibrated do not appear to be so large as to provide a full explanation for how far the simulation model understates household assets and unsecured debts relative to income when compared with the National Accounts.

The discussion reported above indicates that the extent to which the rational agent model understates aggregate unsecured indebtedness in the National Accounts can be largely accounted for by the assumptions that focus attention upon net liquid assets. Including the household asset allocation problem in the rational agent framework appears to be important when attempting to capture preferences toward unsecured indebtedness in the personal sector. Furthermore, although these adjustments will also go some way to explaining the understatement by the rational agent model of aggregate net liquid assets described by the National Accounts, they cannot provide a full explanation. To see this, note that re-scaling unsecured indebtedness generated by the simulation model by the ratio of gross to net household unsecured debt reported in the BHPS data adds approximately £100 billion to the simulated population aggregate. This adjustment is a small fraction of the £1,100 billion gap between the simulated and National Accounts aggregate for net liquid wealth (after adjusting for proportional differences in disposable income).

Khoman & Weale (2006) report that wealth held for bequest purposes was around £2,000 billion in 2003/04. Discounting back to 2001 at the assumed rate of wage growth (1.9% per annum) gives a figure of £1,930 billion. An explicit bequest motive is not included in the rational agent model, though accidental legacies are generated due to the assumption that the time of death is uncertain. The aggregate value of these legacies in the simulated cohort is £1.25 billion. Hence the inclusion of an explicit bequest motive could fully account for the under prediction by the model of aggregate liquid net assets described by the National Accounts.

Hence, the simulation model calibrated to the full UK population appears to capture age trends observed for the incidence and magnitude of debts reported in survey micro data. However, it tends to understate the magnitude of net debts per debtor as described by micro data, and the aggregates reported by the National Accounts for unsecured debt and liquid assets. These differences are largely accounted for by adjusting for coincident holdings of liquid wealth and unsecured debt by households, and for an explicit bequest motive. We now describe in some detail savings and indebtedness implied by the rational agent model calibrated to the full population, and associated underlying motives.

Motives that underly behaviour toward saving and indebtedness

The characteristics reported above provide an indication of the incidence and extent of indebtedness generated by the model. As the micro data regarding financial stocks are considered to be of limited reliability (discussed above), and the current discussion is principally concerned with describing behaviour toward saving and indebtedness implied by the rational agent model, no attempt is made to compare the simulated profiles that are reported below against associated survey data. The profiles that are reported below are consequently not deflated by the assumed wage growth (unlike the preceding discussion, where this adjustment was applied to facilitate comparisons with contemporary survey data).

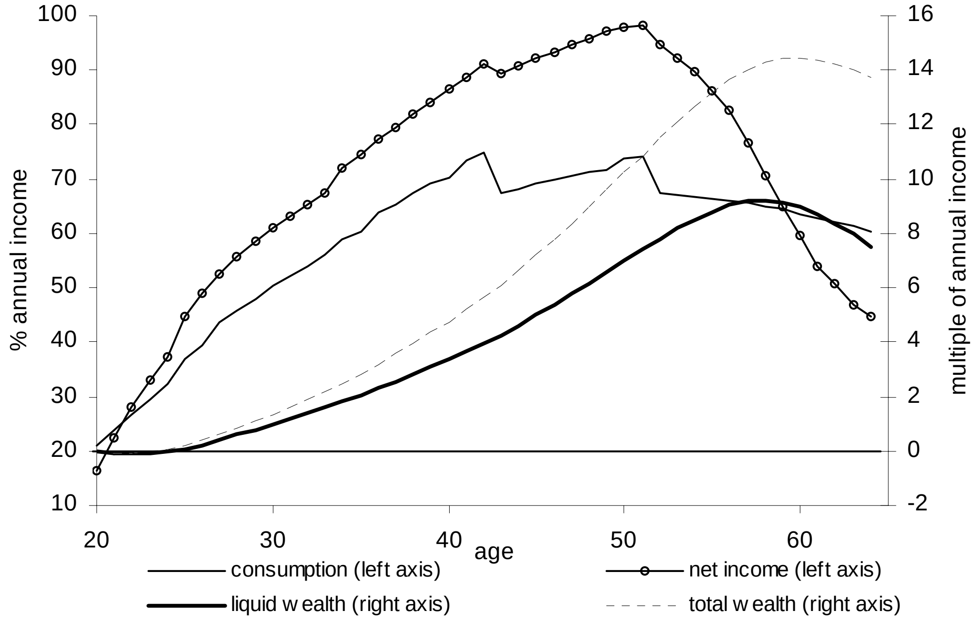
Figure 15 plots age specific arithmetic averages of financial statistics for the full population of 20,000 households simulated using the rational agent model calibrated to the full UK population, and for the population subgroup with some net debt defined on the basis of total wealth (including pension rights) at age 40.

The top panel of Figure 15 reveals that, between ages 20 and 21, average income of the full simulated cohort exceeds average consumption. As households are considered to start out at age 20 with no liquid wealth, most are identified as holding some net debt at age 21. Note, however, that from age 23 to age 59, average disposable income exceeds average consumption, so that the incidence of indebtedness falls off rapidly. These effects underly the incidence of indebtedness reported in Figure 13.

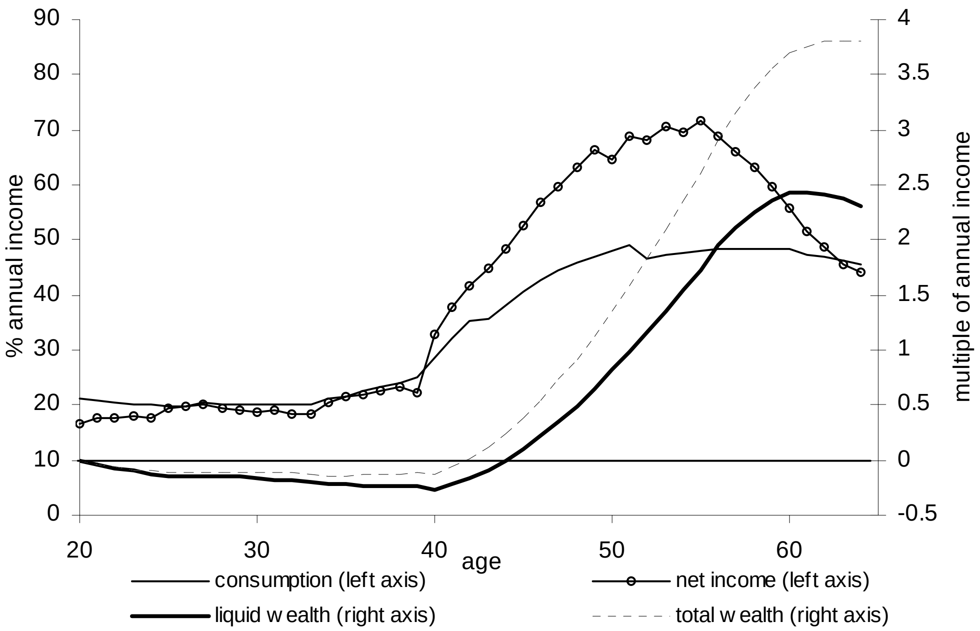
The rate of wealth accrual for the full population is limited early during the simulated lifetime, due to relatively low incomes and the presence of dependent children. Consumption needs fall substantially at age 43 with the departure of children from the first birth issue, and then again at age 52 with the departure of the second issue. The accrual of wealth is most rapid between ages 43 and 52, when some of the dependent children are considered to have left the household while at the same time disposable incomes remain high. Total wealth (including pension rights) peaks at age 60, at approximately 14.5 times average annual gross household income, after which it falls away as households choose to retire and labour incomes are considered to fall.

The age profiles reported for the population subgroup with some net debt at age 40 stand in stark contrast to those for the wider population. The most striking difference is in the profiles reported for disposable income, where the population with some net debt experience a very low and flat age profile to 39, whereas the remainder of the population enjoy substantial income growth. During this period, the average consumption of indebted households tends to exceed disposable income very slightly, so that a growing average debt is accumulated. From age 40 – the threshold age that is used to distinguish the population subgroup – labour incomes of indebted households are seen to grow substantially with age, though they never catch up with the incomes of the wider population. This last observation is due to the returns to savings that the remainder of the population enjoy. The indebted households retire later, so that total wealth continues to grow right up until age 65. At its peak, wealth is just under 4 times average annual gross income; less than a third of the average wealth held by the wider population.

The financial difficulties that are described in Figure 15 for the population subgroup that is net indebted at age 40 are driven by the stochastic processes that are considered to describe family formation and the dynamics of labour income. Households are considered to be perfect financial planners, so that financial capability and access to financial advice have little role to play. It is of note in this context that, even as their financial circumstances deteriorate, it remains sensible for indebted households to continue to spend more than their disposable incomes, right up until the time when their labour market circumstances improve (in this context age 39 by construction). This is because households with low incomes can expect higher incomes in the future under the model, and because welfare is maximised by



Panel B: life profiles of all households



Panel B: life profiles of households with some net debt at age 40

Notes: Age profiles generated from model, using calibrated parameters reported in Table 4

Net debt defined with respect to total household wealth (net liquid wealth plus value of accrued pension rights)

Monetary values expressed as percentages of average annual gross household income between ages 20 and 64, equal to £45,757 in the simulations

Figure 15: Life Profiles of Financial Statistics Generated by Model Calibrated to the Full UK Population

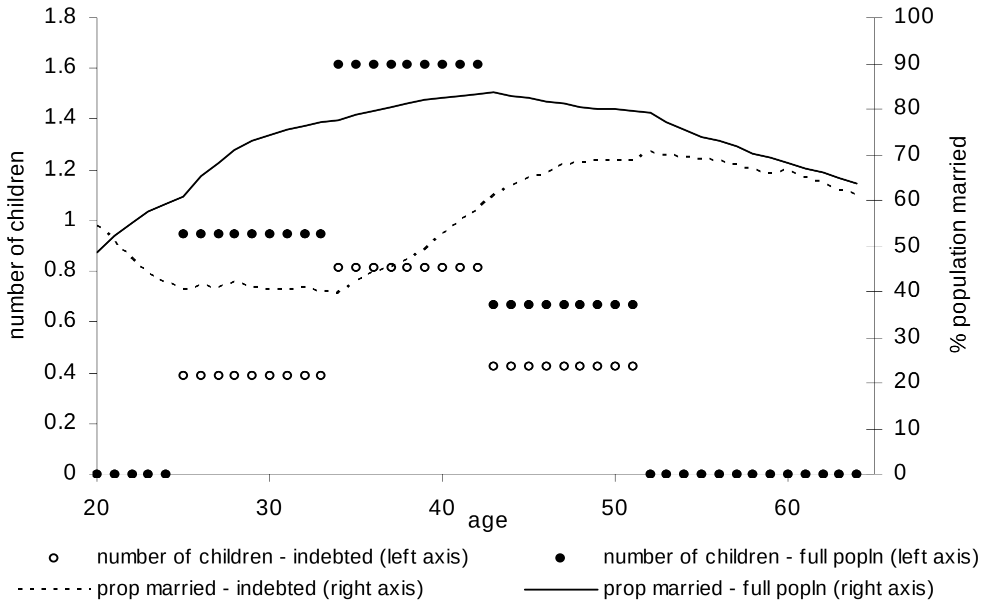
smoothing consumption intertemporally.

Three key factors drive the disposable income profiles reported in Figure 15: labour decisions, the probability of a low wage offer (involuntary unemployment), and household demographics. Age profiles for these statistics are reported for both the indebted population subgroup and the entire simulated cohort in Figure 16.

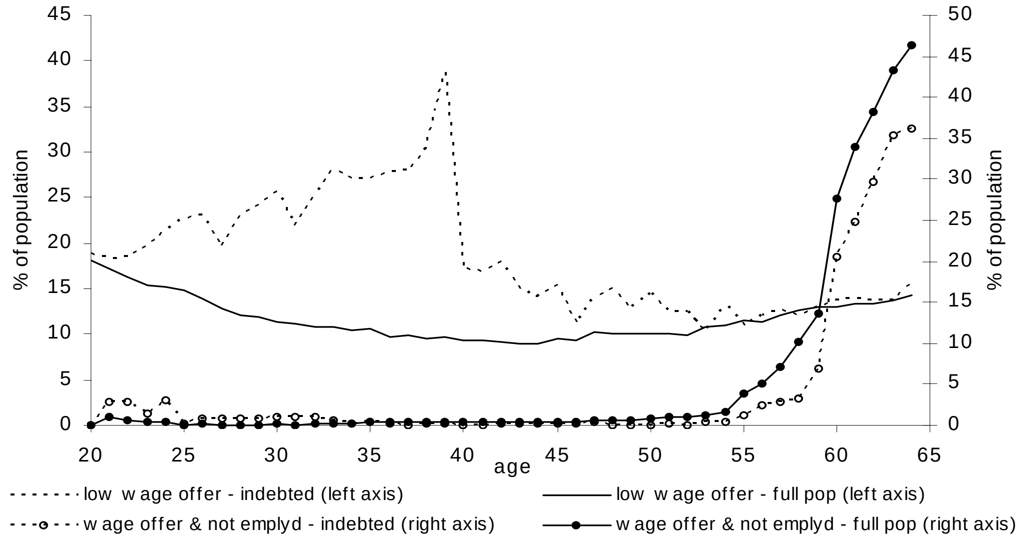
The statistics reported in Figure 16 reveal some striking differences between the simulated life-experiences of households that are identified as net indebted at age 40, and the experience of the wider simulated cohort. The figure suggests that much of the indebtedness generated by the rational agent model can be traced back to variation in marital status. To simplify analysis, demographics in the model are specified so that they are completely exogenous – the equations used to simulate marriage, separation, and child birth do not depend on any household characteristics over which households are considered to exercise some control. Panel A of Figure 16 indicates that the indebted population subgroup begins life with a slightly higher proportion married than the full simulated cohort. In contrast to the full simulated cohort, however, the first five years of the simulated life see a greater number of marital dissolutions than new unions in the indebted population subgroup. Furthermore, the proportion of the indebted subgroup that is married remains substantially below that of the wider population until age 50, with the difference peaking at age 34 – the second child transition period.

Marital status, as considered in the model, influences the financial circumstances of a household in a number of ways. Single households are considered to have a higher probability of receiving a low wage offer, which is one of the factors driving the associated statistics reported in Panel B of Figure 16. The wage generating processes considered for the model imply that single adults on average obtain less income in return for their labour effort when a wage offer is made. Furthermore, single adults are less likely to have children than couples in the model. This influences household behaviour toward saving and indebtedness in two off-setting ways. On the one hand, consumption needs are considered to rise with children, thereby limiting incentives to save. On the other hand, the existence of children imply a rise in disposable income for any given level of private income. In the simulations, the second of these two effects tend to dominate at low incomes (the tax and benefit system provides a larger increase in disposable income than the rise in consumption needs embodied by the assumed equivalence scale), so that indebtedness is more prevalent amongst households without dependent children.

It is important to note, however, that this conclusion – which stands in contrast to empirical incidence of poverty in the population – should be treated with caution for at least three reasons. First, the McClements scale is based upon nutritional needs and not upon actual consumption patterns of the population. Second, our analysis assumes full take up of all welfare benefits, which departs from the practical reality. And third, the intertemporal processes that describe demographic variation in



Panel A: household demographics



Panel B: employment statistics

Notes: Age profiles generated from model, using calibrated parameters reported in Table 4

Net debt defined with respect to total household wealth (net liquid wealth plus value of accrued pension rights)

Figure 16: Simulated Life Profiles of Demographic and Labour Statistics

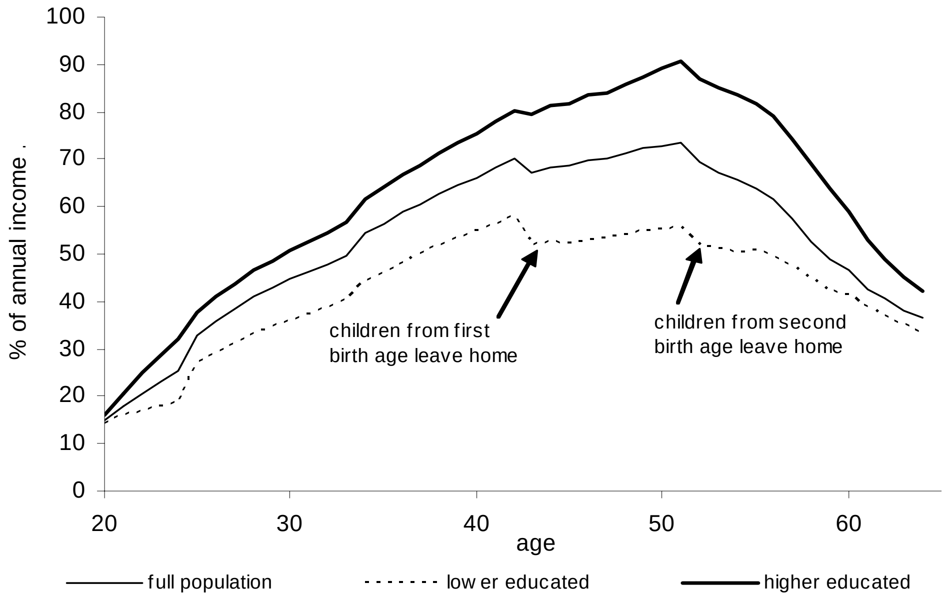
the model do not take into consideration household financial circumstances, whereas empirical studies typically reveal that divorce is negatively correlated with household income, and that single parents often command lower wage rates than single adults without children – particularly at younger ages.

Marital status not only influences the immediate financial circumstances of a household – through labour incomes and consumption needs – but also affects household expectations and provisions for the future. This is because marital status is considered to exhibit considerable intertemporal persistence. At age 20, for example, a married household has a 50% probability of still being married by age 38. In view of this persistence, marital dissolution represents both a negative shock to a household's immediate financial circumstances, and a substantial downward revision of lifetime prospects. As a result of the negative shock, households that suffer a marital dissolution early in the simulated life tend to be worse off than households that were never married. This effect is reflected in Figure 16 by the relative prevalence of marital dissolutions in the indebted population cohort between ages 20 and 25.

The simulated statistics that are reported here consequently emphasise the importance of demographic transitions in driving financial difficulty, an observation that is supported by the empirical research discussed in Section 2. If the assumptions upon which the rational agent model is based provide a reasonable description of the practical reality, then it would appear that unemployment, and access to labour market opportunities have a 'first-round' or 'impact' effect on indebtedness. Underlying these, are the demographics that drive marital status and child birth. The analysis consequently suggests that any policy directive that is designed to tackle private sector indebtedness should take a broad perspective that includes those life transitions that – while not of an explicit financial nature themselves – nevertheless have an important underlying influence on the financial circumstances of households.

7.2 Population subgroups distinguished by education

The analysis of survey data reported in Section 2 reveals a positive relation between the level of education in a household and the debt that is taken on at young ages. In that section we speculated that this was attributable to the respective income profiles of households distinguished by level of education: the hump-shaped profile of disposable income is more pronounced for higher educated households, motivating the accrual of larger debts early during the working lifetime. We explore this hypothesis here, using specifications of the model that are recalibrated to match income profiles for two population subgroups distinguished by their respective levels of education (discussed at length in Section 4.4). In the following discussion, households simulated under the parameters calibrated to sample statistics for the higher educated population subgroup are referred to as higher educated households, and similarly for lower educated households.



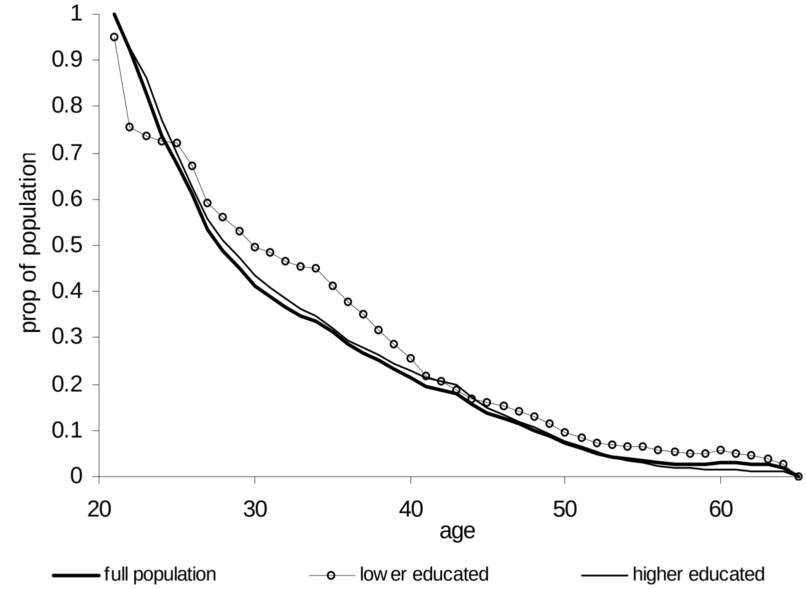
Notes: Simulated statistics – age profiles generated from model, using calibrated parameters reported in Tables 4 and 5
Monetary values expressed as percentages of average annual gross household income simulated between ages 20 and 64 under under the parameters calibrated to the full UK population, equal to £45,757 in the simulations

Figure 17: Average Net Income of Simulated Households by Age and Education Level

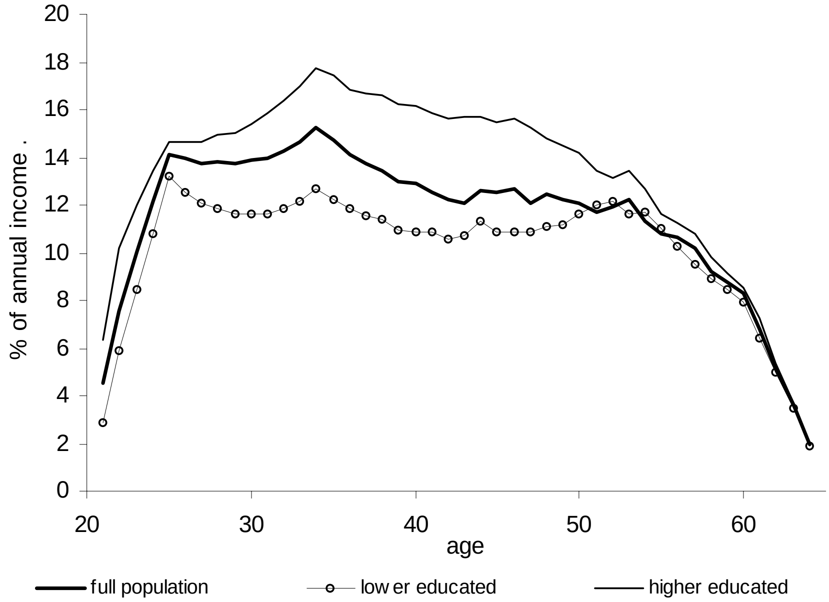
Figure 17 compares the income profiles generated for the three alternative model calibrations that are considered in the current study. These profiles reveal the extent to which the income variation by age of higher educated households exceeds that of lower educated. Specifically, the incomes of higher educated households tend to be similar to those with less education at the beginning of the simulated lifetime. This is attributable in part to ongoing study undertaken by some higher educated households. The incomes of higher educated households tend to rise more rapidly with age than those of the lower educated, so that the average net income of higher educated households exceeds that of the lower educated by the largest margin at age 52, equal to 35% of average annual full-time employment income simulated for the full UK population. The disparity between the two population subgroups then falls away toward retirement, in part because the higher educated tend to be more wealthy and are therefore better able to afford early retirement.

As noted in Section 2, the differences observed between the income profiles of higher and lower educated households have important implications for preferences toward indebtedness implied by the rational agent model. Statistics of indebtedness (measured with respect to net liquid wealth) generated by the rational agent model for the respective population subgroups are reported in the two panels of Figure 18.

The statistics that are reported in Figure 18 reflect the observations derived from BHPS survey data discussed in Section 2. Specifically, the simulation model generates a lower incidence of indebtedness,



Panel A: proportion of population indebted



Panel B: average debt per debtor

Notes: Simulated statistics – age profiles generated from model, using calibrated parameters reported in Tables 4 and 5
Monetary values expressed as percentages of average annual gross household income simulated between ages 20 and 64 under
under the parameters calibrated to the full UK population, equal to £45,757 in the simulations

Figure 18: Indebtedness by Age and Education – simulated data

and smaller debts on average between ages 21 and 24 for the lower educated population subgroup than it does for the higher educated. Although the incidence of indebtedness is higher for lower educated households between ages 25 and 65, it is not until age 50 that the aggregate indebtedness simulated for lower educated households exceeds that of a similar sized population of higher educated households.

It is worth taking a moment to explain the key reasons for the differences observed between the simulations distinguished by education level that are reported here. As discussed previously, an agent in our model will rationally take on debt if they expect that income relative to consumption needs will rise in the future. There are two key reasons why an agent will expect higher income in the future under our model. The first is the age trend that is assumed for labour income, which agents are considered to take into account in forming their expectations regarding the future, and the second is uncertainty from one year to the next, which gives rise to good years and bad. The income profiles reported in Figure 17 indicate that the age trend increases more steeply early during the simulated lifetime for higher educated households than for the lower educated. In contrast, the parameter values that are reported in Section 4.4 indicate lower educated households are exposed to greater income uncertainty in that they have a higher probability of receiving a low wage offer in any given year.

Stronger income growth early in the working lifetime, and larger incomes in absolute terms, motivate the accrual of larger debts amongst the higher educated. In contrast, higher probabilities of a low wage offer motivate a higher incidence of indebtedness amongst the lower educated. It is of particular note that the first of these two effects dominate the relation between the age specific aggregates for unsecured debt of the two population subgroups to age 50, by which time aggregate indebtedness has fallen substantially for both population subgroups.

Hence, relative to lower educated households, higher educated households tend to accrue debt in the model to maintain high consumption at the beginning of their lifetimes in response to their expectations regarding income growth. Lower educated households, by contrast, tend to accrue debt in response to the experience of an adverse shock. It would seem advisable to facilitate access to credit in both of these circumstances. It is, however, likely that the latter will require a more active role from government than the former.

If the above observations provide a qualitatively accurate representation of the practical reality, then the trend toward a higher educated population should result in greater demand for credit early in any given birth cohort's lifetime, and larger debts for those who are indebted. If the provision of credit to the young is perceived to be profitable, then there may be little need for government intervention to facilitate the above transition. If, however, credit providers are only willing to offer credit at reasonable rates to young individuals who provide some form of capital guarantee (in the form, for example, of a parental guarantor), then it may make sense from an equity perspective for the government to intervene in the market. This intervention could take the form of a student loans scheme.[53]

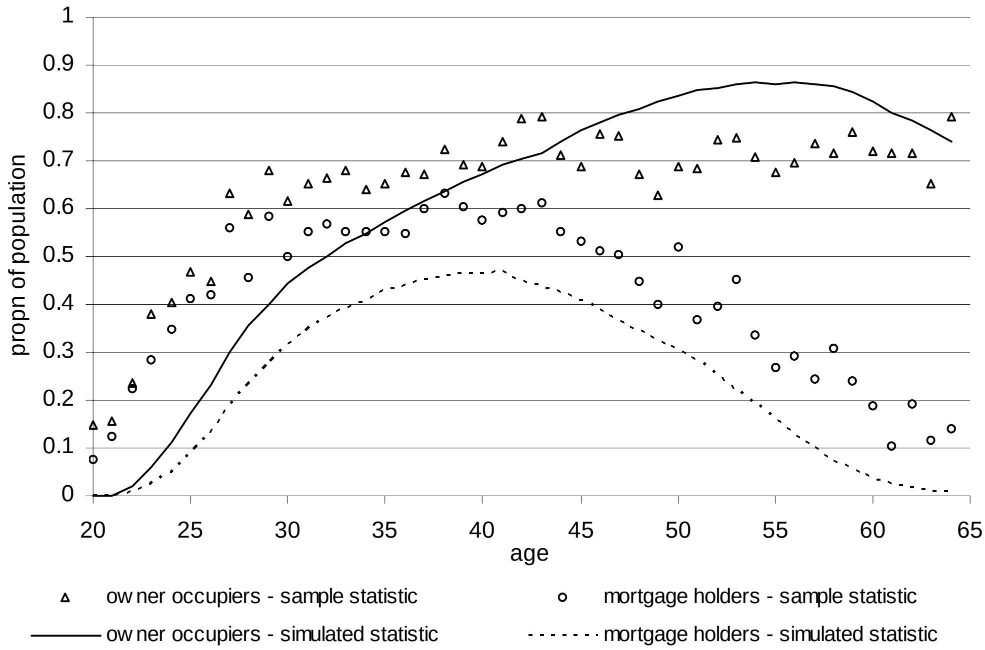
7.3 Housing and mortgage debt

The discussion reported in Section 2 reveals that the vast majority of debt taken on by the private sector is used to purchase real estate. The current section reports the results of projecting wealth held in residential housing and mortgage debt for the simulated cohort calibrated to the full UK population. The measures of housing wealth and mortgage debt reported in this section were generated using the econometric models described in Section 5. This *reduced form* approach is an important analytical departure from the *rational agent* approach that was adopted to generate labour supply, consumption, net liquid assets and unsecured indebtedness, reported elsewhere in the study. The rational agent approach involves selecting individual decisions to maximise expected lifetime utility, given assumptions about the future that are consistent with the processes by which individual specific circumstances are considered to evolve. In contrast, the reduced form approach is designed to project housing wealth and debt to capture statistical correlations between those characteristics and characteristics endogenous to the rational agent model, as described by cross-sectional survey data reported in the 2000/01 wave of the BHPS. *It is important to bear this in mind when interpreting the statistics that are reported below.*

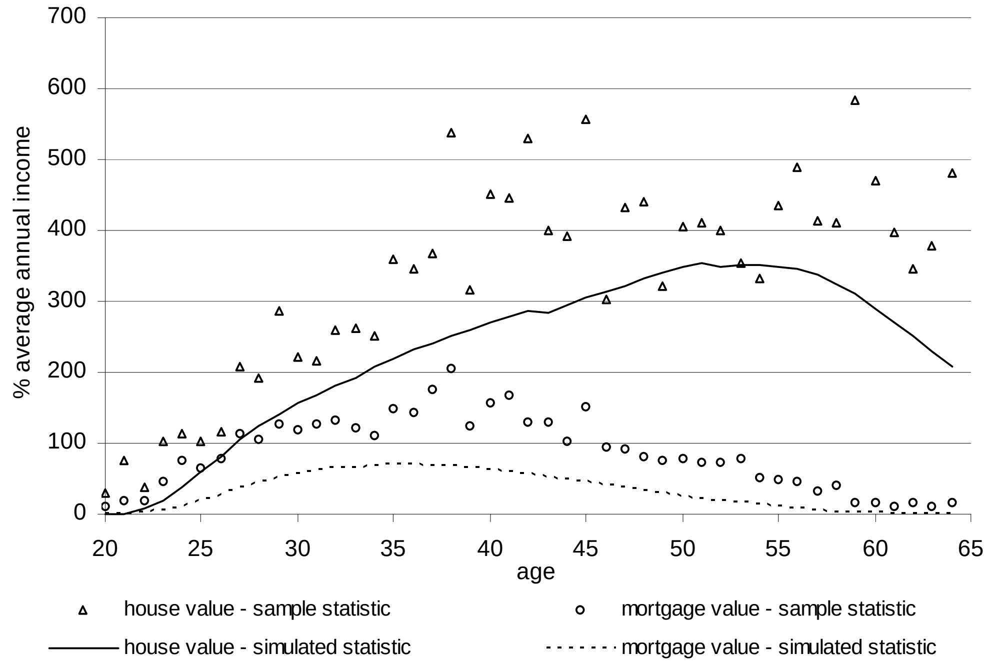
Figure 19 reports some key statistics for housing and mortgage debt. The top panel of the figure indicates that the projections considered for the simulations do a reasonable job of capturing the age specific trends described by BHPS survey data for the incidence of both owner occupied housing and mortgage debt. However, the statistical projections tend to understate the level of the age trends, particularly in the case of the incidence of mortgage holders, which understate the sample statics by around 15% for much of the period between ages 20 to 65. These observations are, in part, attributable to the assumption in the model that households all enter the model at age 20 with zero wealth – an observation that is approximately true on average (see Figure 12 for associated survey statistics), but does not describe the practical reality for the entire population.

Similar observations to those described for the incidence of housing and mortgages also apply for the average values reported in the bottom panel of Figure 19. This is not surprising as the bottom panel of the figure reports aggregate population averages rather than averages across the population subgroup identified as householders – if the projections for the values of housing wealth and mortgage debt contingent on incidence provided a perfect reflection of the survey statistics, then the same proportional disparities would be reported in the two panels of Figure 19. Hence, as in the case of the disparities observed between the simulated and sample statistics for incidence, much of the disparity between the

[53] There are various models that could be adopted for the specification of student loans. One interesting model is the Higher Education Contribution Scheme that is currently administered in Australia. This scheme involves the provision of credit to tertiary students. The debt is indexed to inflation, and is repaid in the form of a higher rate of income tax charged when taxable income exceeds a certain threshold.



Panel A: proportion of households owner occupiers / mortgage holders



Panel B: population average house and mortgage values

Sample statistics: author calculations using BHPS survey data

expressed as percentages of average annual gross household income for ages 20-64, equal to £23,208

Simulated statistics: author calculations using simulated data, based on calibrated parameters reported in Table 4

age specific statistics discounted for 1.9% annual wage growth

expressed as percentages of average annual gross household income for ages 20-64, equal to £30,487

Figure 19: Housing and Mortgage Statistics by Age – simulated and survey data

statistics reported for value in the bottom panel of Figure 19 can be attributed to the assumption of zero wealth at age 20 for all households simulated by the rational agent model.

7.4 Summary

- The rational agent model implies less unsecured debt and less non-pension wealth in aggregate than reported by the National Accounts.
- The rational agent model overstates both the rate of incidence and aggregate value of unsecured debt described by BHPS micro data.
- Taking the National Accounts data as the appropriate basis for comparison, the understatement of aggregate indebtedness by the simulation model is substantially explained by the understatement of disposable household income (due to conceptual difficulties associated with calibrating the model), and by assumptions that simplify the household asset allocation problem. Nevertheless, adjusting for these effects still implies that the simulation model understates aggregate unsecured debt reported in the National Accounts by 30%.
- In contrast, the omission from the rational agent model of an explicit bequest motive could reasonably explain the understatement of aggregate non-pension wealth reported in the National Accounts.
- Indebtedness in the model arises primarily as a product of low incomes. Underlying low incomes are marital status and the (exogenously imposed) probability of a low wage offer, which tend to have a more pronounced influence on indebtedness following a marital dissolution early in the simulated lifetime. These observations are qualitatively consistent with survey data, and emphasise the importance of "indirect effects" as drivers of private sector indebtedness.
- The model suggests that the trend toward a higher educated population should result in greater demand for credit early in any given birth cohort's lifetime, and larger debts for those who are indebted. This suggests that there may be a larger role to play for student loan programmes.

8 The Incidence of Poverty and Over-Indebtedness

The analysis reported above suggests that, subject to the assumptions embodied by the rational agent model, most households should choose to be net debtors at some point in their lives. We focus here upon the related issues of poverty and over-indebtedness, which arise in the model as a product of the uncertainty that characterises the intertemporal evolution of individual specific circumstances – even perfect financial planners can suffer periods of poverty when they are exposed to risk.

The current analysis focuses upon two related questions: what defines the characteristics of households that are likely to become over-indebted or impoverished in the near future, and what is the likelihood that such characteristics will describe households that behave rationally throughout their lives? The first of these questions is an issue that depends predominantly on the processes that are assumed to govern the evolution of household circumstances. The second question, in contrast, depends heavily on the choices that households make throughout their respective lifetimes. We begin by clarifying some definitions for analysis, before exploring each of the questions referred to above.

8.1 Some definitions

Over-indebtedness could usefully be interpreted as the condition in which an agent has insufficient financial means to comfortably service their existing loans. This definition of over-indebtedness is far from precise. A broad interpretation of 'financial means', for example, would take into account all of the aspects that influence the financial circumstances of an agent, including expectations regarding future income flows and consumption needs. Furthermore, how should we define a threshold beyond which debt servicing becomes 'uncomfortable', and should the servicing of existing loans take into account repayment of the principal?

The Consumer Credit White Paper (DTI (2003)) considers an individual to be over-indebted if their unsecured debt servicing accounts for at least 25% of their income net of welfare benefits but gross of taxes. We adopt a variant of this definition for the current study, interpreting 'debt service' as the interest charge on unsecured loans, and making comparisons against total disposable income (net of welfare benefits *and* taxes), so that:

> A household is considered to be over-indebted if the interest charge on their unsecured loans exceeds 25% of their total disposable income.

This definition has the advantage that it is simple and transparent. It does not, however, take into account expectations, and perhaps more importantly, does not reflect associated consumption needs.[54] To appreciate this last point, note that the assumed definition for over-indebtedness will imply that any two households with the same disposable incomes and the same unsecured debts will receive the same classification under the above definition of over-indebtedness, regardless of their respective demographic circumstances. As a household that is comprised of a single adult can be assumed to have fewer consumption needs than another that is comprised of two adults and three children, this

[54] The difficulty associated with measuring over-indebtedness is highlighted by empirical studies, which suggest that the condition is not adequately described by any single statistic. Recent empirical findings reported by the Department for Trade and Industry (DTI (2005)), for example, reveal that there is little over-lap between alternative indicators of over-indebtedness, and that – of the population subgroup in their survey with unsecured debts over the 25% servicing limit – 58% thought that their repayments were not a problem, and just 9% reported that they considered their repayments to be a heavy burden.

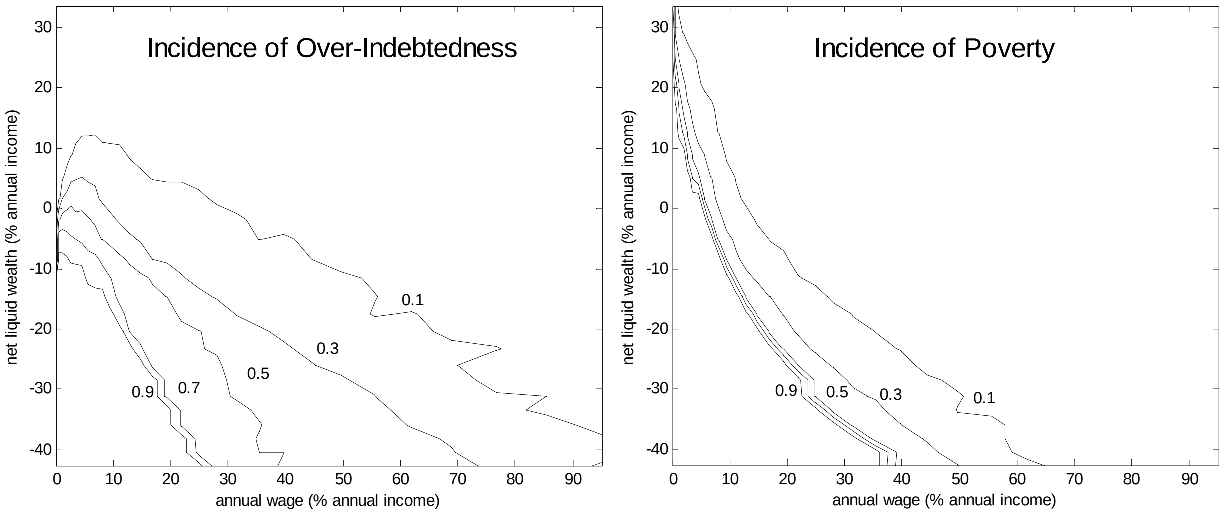
definition of over-indebtedness fails to reflect respective abilities to pay, which might be required of a more comprehensive definition.

The analysis of over-indebtedness as defined above is consequently complemented here by reporting measures of the proportion of households with equivalised consumption (aggregate consumption divided by the equivalence scale) of less than £100 per week. These households are referred to as "poor", reflecting the interpretation of the assumed threshold of equivalised consumption as a measure of deprivation. The poverty statistics provide a useful counterpoint to the measures of over-indebtedness that are referred to above because they capture the influence of household demographics – and consumption needs in particular – on welfare. The threshold rate of £100 was selected to approximately equate the proportions of the population aged 20 and single that are identified by the model as over-indebted to the proportions identified as poor. This is discussed at further length below.

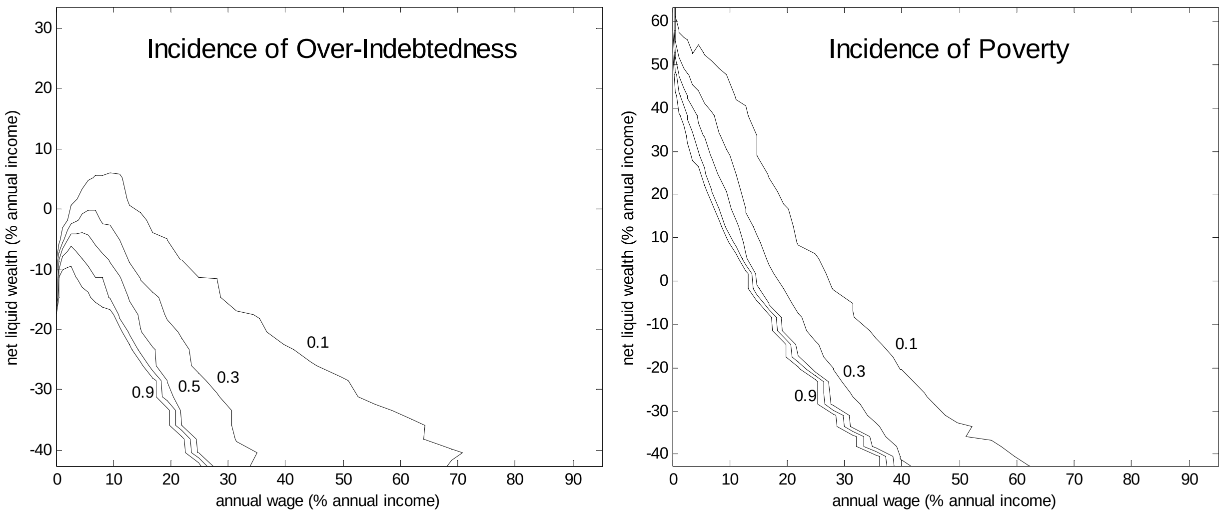
8.2 The characteristics that lead to over-indebtedness and poverty

We begin by considering the probability of poverty and over-indebtedness for households aged 20, which reflects the observation reported in Section 7 that indebtedness is most pronounced early in the simulated lifetime. A two dimensional grid in household wage and net liquid wealth was taken for both singles and couples, assuming zero pension wealth and that a wage offer is received at age 20. The model was then used to project 1,000 households forward four years, from each of the intersecies of the two dimensional grids. These generated data were used to calculate the risk associated with being over-indebted or poor (as the terms are defined in the preceding subsection) in any year between ages 20 and 24, given any combination of household characteristics. The results of this analysis are reported in the form of contour maps in Figure 20.

The statistics reported in Figure 20 indicate that single adults aged 20 are at a higher risk of becoming over-indebted, than otherwise similar couples. In the case of 20 year-old households with net (unsecured) debts and wages both equal to 20% of average annual income, for example, Figure 20 indicates that single adult households have a 60% chance of becoming over-indebted at some time between ages 20 and 24, which is approximately twice the probability identified for couples. This conclusion is consistent with the discussion reported in the Section 7, and is observed despite the fact that couples have higher consumption needs than singles, due primarily to differences in the assumed wage generating processes. First, the labour income of single adults is assumed to follow a lower trend progression, so that singles tend to be poorer from a lifetime perspective than couples. More importantly, however, singles are considered to have a 30% chance of receiving a low wage offer in any year of the working lifetime, whereas couples have a 5% chance. With lower incomes, and more tenuous employment opportunities, single adults experience a higher prevalence of over-indebtedness.



Panel A: single adults



Panel B: couples

Notes: Contour maps generated from model, using calibrated parameters reported in Table 4

Monetary values expressed as percentages of average annual gross household income between ages 20 and 64, equal to £45,757 in the simulations

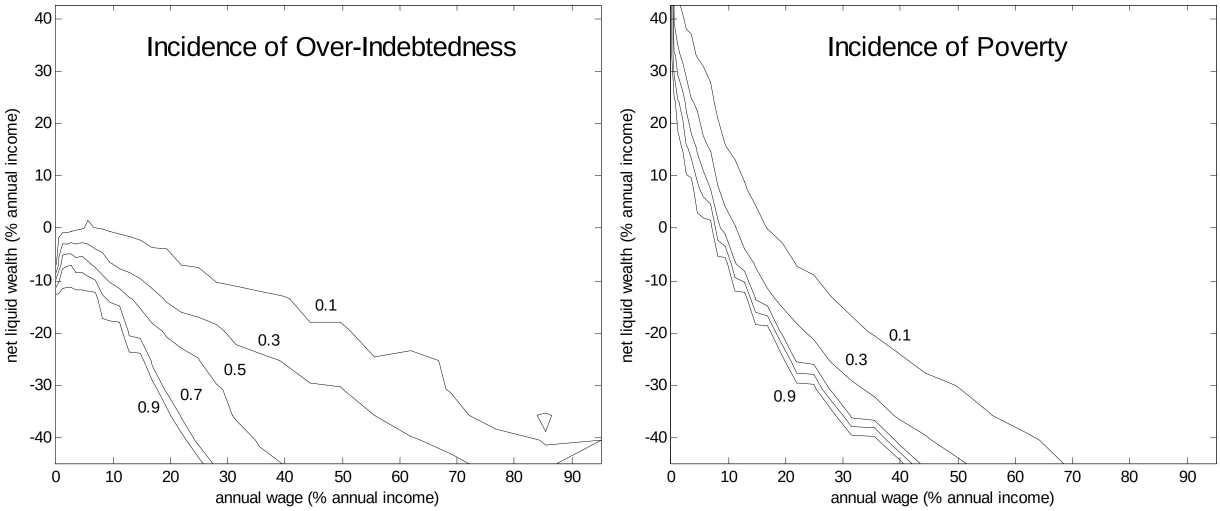
Figure 20: Probability of Being Over-Indebted or Poor in at least One Year Between Ages 20 and 24 – contour maps by wage and wealth at age 20

The profile of the contours reported for over-indebtedness in Figure 20 provide an interesting incite into household behaviour with regard to debt. Consider, for example, the circumstances of a single adult household aged 20 with a wage of £0, and an unsecured debt equal to 15% of average annual income. The top panel of the figure indicates that a household with these circumstances has a 100% probability of being over-indebted during at least one year between ages 20 and 24 – an observation that arises because it is defined as over-indebted at age 20 by assumption. Now consider two alternatives for this household: giving it a once-off lump-sum payment equal to 25% of average annual income at age 20, or giving it a wage equal to 25% of average annual income at age 20. The latter adjustment is clearly substantially more generous than the first, due to the intertemporal persistence that is assumed to describe the wage generating process. Nevertheless, the left-hand side of Panel A of Figure 20 indicates that the lump-sum gift results in a larger drop in the probability of over-indebtedness.

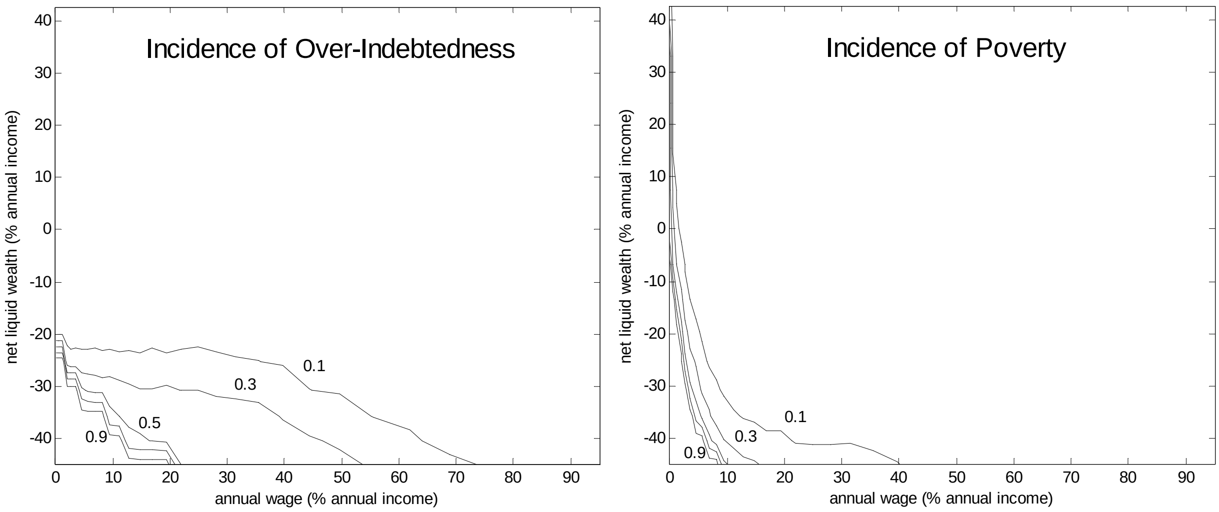
This apparently perverse observation arises due to the rational reactions predicted by the model to the two alternatives that are considered. In the example described above, increasing the household's wealth only affects the lifetime resources that they have for allocation. As the transfer is sufficient to provide positive net worth at age 20, the household will simply choose to consume a fraction of the wealth at age 20, and at all future periods of the considered lifetime. In contrast, increasing the household's wage increases both their exposure to uncertainty and their expectations regarding future financial circumstances. On the one hand, uncertainty tends to encourage households in the model to undertake precautionary savings, and on the other an increase in expected financial circumstances for the future encourages increased current consumption; a balancing act that is governed by the degree of risk aversion that is assumed for individual preferences. Figure 20 indicates that the combination of increased uncertainty and the motive to increase current consumption to reflect the improvement in expected lifetime resources give rise to a larger probability of over-indebtedness than a simple lump-sum transfer.

The impact of consumption needs on the financial circumstances of singles and couples is made clear by the contour maps that are reported for the incidence of poverty in Figure 20. Although the two contour maps reported for poverty in the figure look very similar at first glance, note that the vertical scale for couples extends to twice the upper bound that is reported for singles. Hence, couples aged 20 and with low wealth and wages are at a higher risk of experiencing at least one year of very low equivalised consumption than similarly placed singles.

Figure 21 reports similar contour maps as those in Figure 20, but for single adults between ages 25 and 29. Panel A of the figure displays contour maps for single adults without any dependent children, which – unsurprisingly – are very similar to those reported for single adults in Figure 20. Panel B, in contrast, reports contour maps for single adults with one dependent child. Comparing Panels A



Panel A: single adults with no dependent children



Panel B: single adults with one dependent child

Notes: Contour maps generated from model, using calibrated parameters reported in Table 4

Monetary values expressed as percentages of average annual gross household income between ages 20 and 64, equal to £45,757 in the simulations

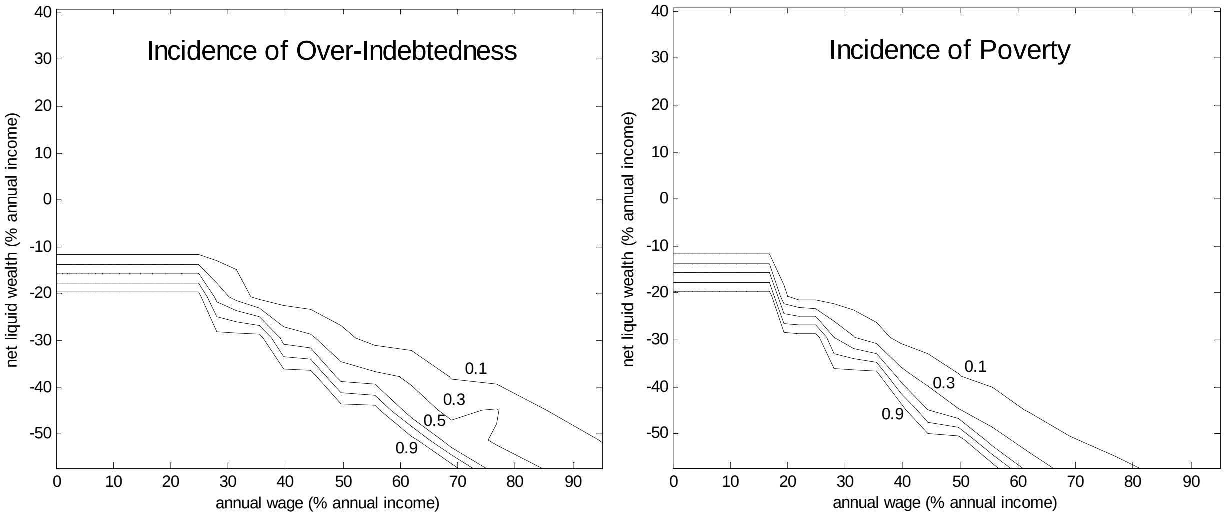
Figure 21: Probability of Being Over-Indebted or Poor in at least One Year Between Ages 25 and 29 – contour maps by wage and wealth at age 25

and B of Figure 21 indicates that the probability of over-indebtedness and poverty tend to be lower for households with dependent children than those without, a conclusion that is not particular to the pairwise comparison that is provided here (see Appendix D for further statistics). The result arises because the tax and benefit system that is considered for analysis makes a larger adjustment in response to the presence of a child than the increase in consumption needs that is implicit in the (McClement's) equivalence scale that is assumed for the preference structure. This observation is perhaps unsurprising from a policy perspective, given the effort that has been made during the last decade to improve the financial circumstances of households with dependent children. It is important to note, however, that the result depends heavily upon the value judgements that are implicit in the assumed equivalence scale. Different results would be obtained if the relative consumption needs of children in practice are higher than their nutritional requirements upon which the McClements scale is based, or if take-up rates for associated benefits were under 100%. Furthermore, in relation to the scatter plots reported in Figure 21, the fact that the model takes no account of financial circumstances in generating temporal dynamics of demographics is likely to be particularly important for household with dependant children.

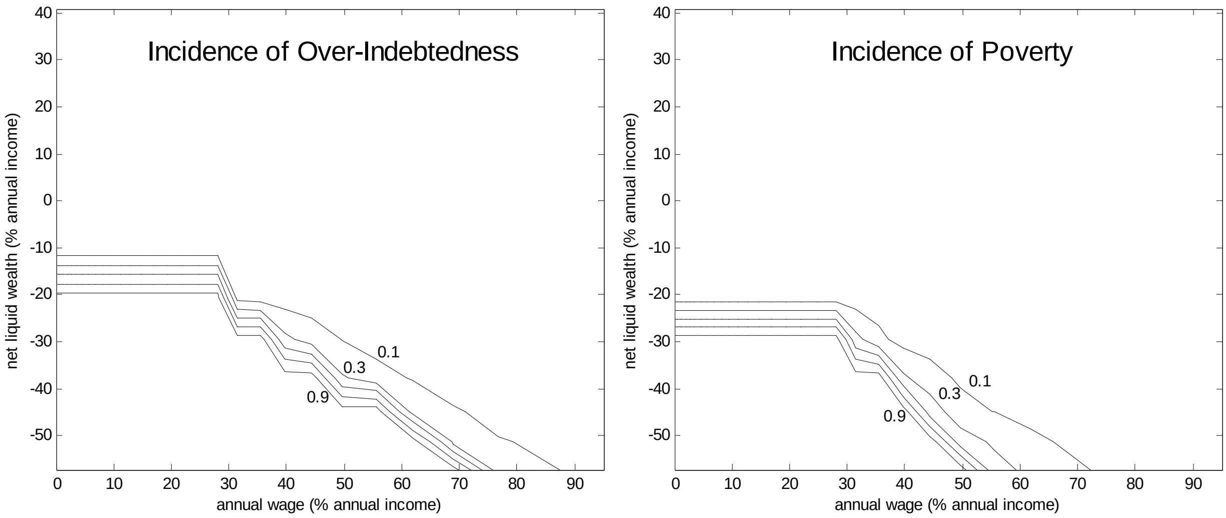
We conclude this subsection by considering the circumstances of households as they near state pensionable age (65 in the simulations). State pensionable age is interesting with reference to the rational agent model, because it is assumed that all unsecured debt must be repaid by this time. Hence, as state pensionable age nears in the model, the maximum unsecured debt that a household may have falls, from just under £31,000 at the beginning of age 55 to £7,100 at the beginning of age 64 (see Table 4 for full details). In these circumstances it seems reasonable to include repayment of the principal when defining over-indebtedness:

> A household is considered to be over-indebted if the interest charge on their unsecured loans, plus the fraction of the principal that is inversely proportional to the number of years before they reach retirement age exceed 25% of their total disposable income.

Figure 22 reports contour maps for the probability that a 60 year old household is over-indebted (as it is redefined here) or impoverished (as it is defined in the preceding subsection) in any year between ages 60 and 64. All four of the contour maps reported in Figure 22 describe very similar profiles, suggesting a closer relation between the measures of poverty and over-indebtedness considered for 60 year olds, than those considered for 20 year olds in Figure 20. This is to be expected, given that measures of indebtedness have a more immediate impact on consumption toward state pensionable age, due to the assumption that all unsecured debt must be repaid by that time. Furthermore, individuals are eligible for welfare benefits from age 60 that are worth almost twice the benefits that are provided to childless households at age 20, which eliminates poverty for households with small debts and very



Panel A: single adults



Panel B: couples

Notes: Contour maps generated from model, using calibrated parameters reported in Table 4

Monetary values expressed as percentages of average annual gross household income between ages 20 and 64, equal to £45,757 in the simulations

Figure 22: Probability of Being Over-Indebted or Poor in at least One Year Between Ages 60 and 64 – contour maps by wage and wealth at age 60

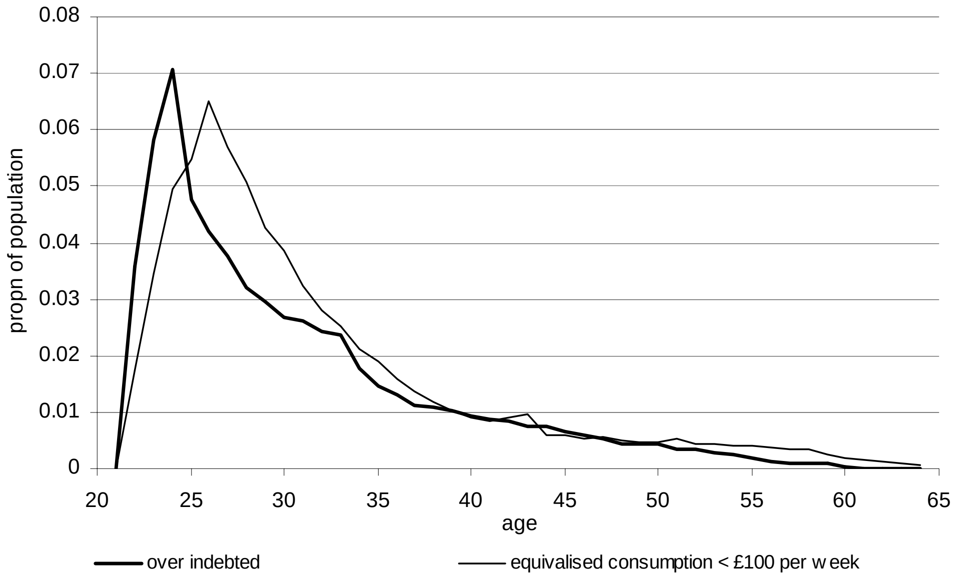


Figure 23: Proportions of Population Over-Indebted / Credit Constrained by Age

low annual wages. This last observation highlights the practical importance of the welfare judgement implicit in providing the pension guarantee from age 60.

8.3 The likelihood of becoming over-indebted when behaving rationally

To determine the likelihood of being constrained when behaving rationally, the current discussion is based upon the cohort of 20,000 households that was generated by the rational agent model calibrated to the full population (as discussed at length in Section 7). Analysis of this population revealed that 22% of generated households were over-indebted during at least one year of their simulated lives, and that 19% chose to consume less than £100 per week (in equivalised terms) during at least one year between the ages of 20 and 75. Furthermore, 1.4% of simulated households spent at least one year during the working lifetime with less than £500 of available credit. These statistics indicate that credit constraints play an important role in determining the consumption behaviour generated by our model, which – in itself – is of little surprise.[55] Of greater interest is the timing of the incidence of poverty and over-indebtedness, and the reasoning behind observed events.

Figure 23 plots age profiles for the proportion of the simulated population identified as over-indebted, and the proportion with equivalised consumption under £100 per week. The two age profiles reported in the figure are very similar, which underscores the practical implications of over-indebtedness for household consumption. The figure indicates that the incidence of over-indebtedness is highest at age

[55] As noted in Section 3.6, for example, the credit constraint explicitly determines consumption behaviour at the terminal age adopted for the model.

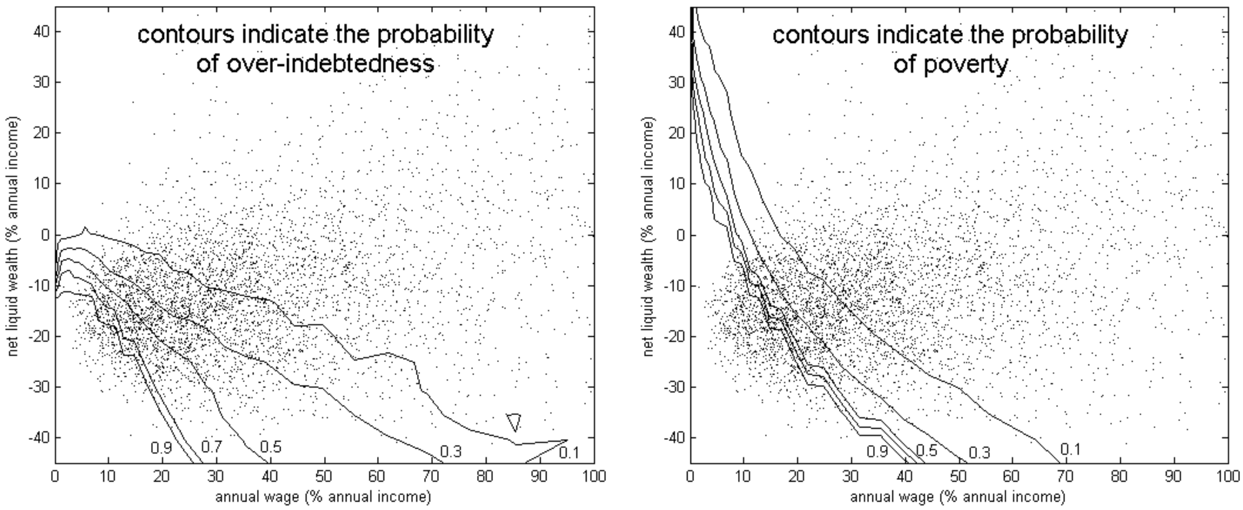
24, just prior to the birth of children in the model, and falls away rapidly after that age. In contrast, the incidence of poverty (as it is defined above), rises sharply at age 25 and exceeds the incidence of over-indebtedness until age 39, by which time the two series have converged.

The discrete fall in the proportion identified as over-indebted at age 25 is due to the higher welfare benefits that are payable to households with dependent children. The coincident discrete rise in the proportion of the population identified with low consumption highlights the fact that the assumed definition for over-indebtedness makes no allowance for consumption needs. The observation that over-indebtedness is concentrated in younger households is consistent with survey data, and reflects the dis-proportionate influence of negative income shocks for the young. This is underscored by the observation that, of those households identified as over-indebted, 74% had suffered a low wage offer in the same year, and 13% had experienced a divorce at some time during the preceding five years. The positive correlation of over-indebtedness with these shocks to personal circumstances is consistent with associated empirical findings, as discussed in Section 2.

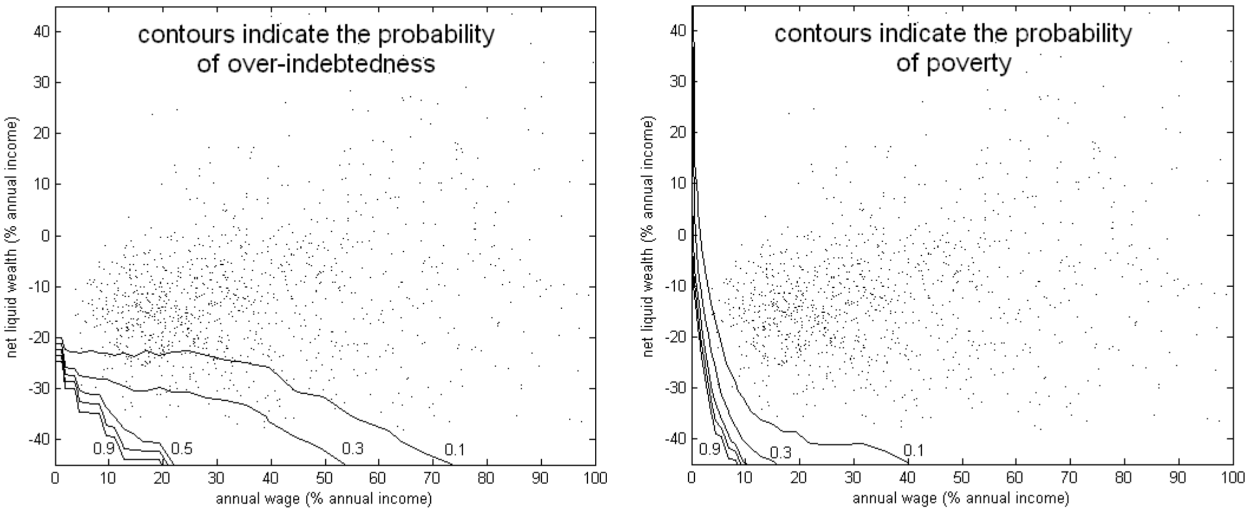
To explore the circumstances that underly the incidence of over-indebtedness and poverty reported above, Figure 24 augments the contour maps reported in Figure 21 to include point markers that indicate the circumstances of those simulated households falling within the domain displayed in the respective graphs. Specifically, of the 20,000 households simulated for analysis, 4,681 were identified as single adults at age 25, 4,384 of whom were identified with net liquid assets and annual wages below the upper thresholds reported Panel A of Figure 24. Similarly, of the 1,198 households identified as single adults with one dependent child at age 25, 1,136 are reported in Panel B of the figure.

Panel A of Figure 24 indicates that a sizeable proportion of single adults without dependent children at age 25 in the model are exposed to a high probability of experiencing at least one year of over-indebtedness and poverty between ages 25 and 29. Indeed, the bulk of the single and childless population are concentrated in a region of net liquid wealth and annual wage that is associated with a 30% probability of over-indebtedness and poverty, and many are exposed to much higher risk of both events. In contrast, Panel B of the figure indicates that very few single adults aged 25 with one dependent child are subject to an appreciable probability of over-indebtedness, and almost none are likely to experience poverty within the 5 year horizon considered here. This should be seen in the context of the caveats that are made in the preceding subsection.

Figure 25 extends the above analysis to 25 year old couples. 4,946 households are identified as 25 year old couples without dependants in the simulated cohort, and 2,679 are identified as couples with one dependent child. In contrast to the case of singles, however, Figure 25 omits a sizeable share of these sub-populations – 3,611 couples without dependent children are identified with net liquid wealth and annual wage falling within the relevant figures, and 1,936 couples with one dependent child. The



Panel A: single adults with no dependent children

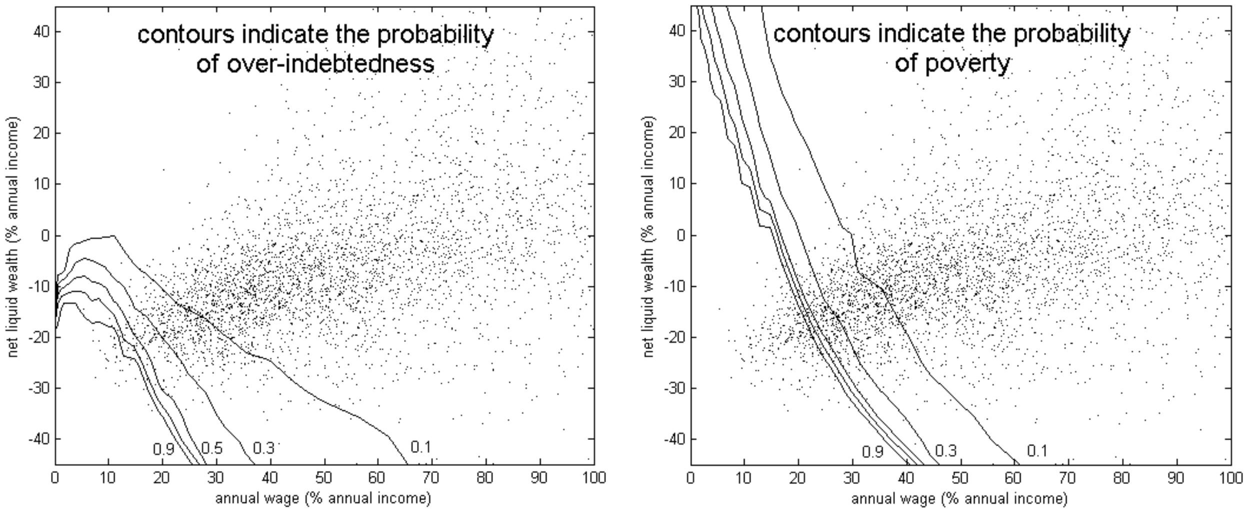


Panel B: single adults with one dependent child

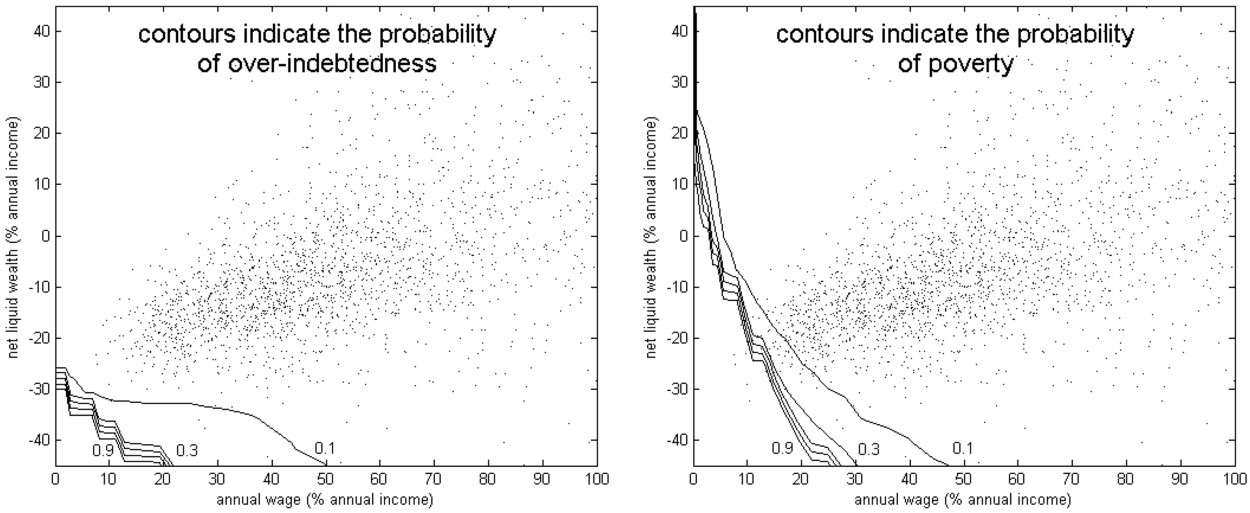
Notes: Contour maps generated from model, using calibrated parameters reported in Table 4

Monetary values expressed as percentages of average annual gross household income between ages 20 and 64, equal to £45,757 in the simulations

Figure 24: Probability of Single Adults Being Over-Indebted or Poor in at least One Year Between Ages 25 and 29 – contour maps by wage and wealth at age 25



Panel A: couples with no dependent children



Panel B: couples with one dependent child

Notes: Contour maps generated from model, using calibrated parameters reported in Table 4

Monetary values expressed as percentages of average annual gross household income between ages 20 and 64, equal to £45,757 in the simulations

Figure 25: Probability of Couples Being Over-Indebted or Poor in at least One Year Between Ages 25 and 29 – contour maps by wage and wealth at age 25

sizeable proportions of the population subgroups that are omitted from Figure 25 emphasise the higher incomes that couples are considered to earn in the rational agent model relative to singles, and provide an intuitive pre-cursor to the observations that are reported here.

Couples in the rational agent model tend to be exposed to a low probability of being over-indebted between ages 25 and 29, with the risk tending to zero with the addition of a dependent child. Furthermore, Panel A of figure 25 indicates that couples without a dependent child are also subject to a low risk of poverty, relative to singles. In contrast to these observations, however, the risk of experiencing at least one year of poverty between ages 25 and 29 appears to be higher amongst couples with one dependent child than it is amongst singles. This observation is driven by the fact that the proportion adjustment made by the tax system in response to a dependent child exceeds the adjustment in household needs (as represented by the equivalence scale), by a wider margin for singles than for couples.

To better understand the behaviour of households that are critically indebted at some point in their simulated lives, Figure 26 reports age profiles for the circumstances of households that were identified to undertake equivalised consumption of less than £50 per week during at least one year of their simulated working lifetime (ages 20-64), who comprise 1.1% of the simulated cohort. There are clear similarities between the critically indebted households identified here, and the profiles for indebted households more generally that are reported in Section 7, which suggests that the statistics reported there are not driven primarily by the thresholds used to define the population subgroup. As discussed in Section 7, all of the profiles reported in Figure 26 indicate that households which suffer sever deprivation in at least one year of their working lives tend to have a life experience from the outset that differs from the remainder of the simulated cohort. Net income starts out lower on average for the poorer subgroup, and trends downward, resulting in a disparity with the remaining population that grows substantially by age 30. The downward trend in income is mirrored by consumption, resulting in an "expenditure crunch" around age 26-27 – just after the first year in which dependent children are considered to enter the model. Thereafter, consumption and income both tend to rise, but at a less rapid rate than observed for the remainder of the population.

Relative to income, consumption begins higher and trends down faster with age for the poor population subgroup, resulting in negative net liquid wealth that grows faster in the early 20s and peaks at age 27. This relation is attributable to two important influences. First, at the beginning of the simulated lifetime, a household with an unlucky draw may bet upon the likelihood of receiving better draws in the future. As each subsequent draw also turns out to be poor, the household will revise its expectations regarding lifetime income downward, and adjust its per-period consumption down accordingly. Second, the steeply inclining interest rates assumed for unsecured debt discourage debt accumulation, particularly for low income households. After age 30, net liquid wealth rises until retirement, but the poor on average never manage to catch up to the savings accrued by the remainder of the population.

8.4 Summary

- The rational agent model implies that 22% of households should experience at least one year of over-indebted during their lives, and that 19% should chose to consume less than £100 per week (in equivalised terms) during at least one year between the ages of 20 and 75. *Over-indebtedness and financial deprivation cannot simply be dismissed as a product of poor financial planning.*

- Over-indebtedness generated by the rational agent model peaks at 7% of the population at age 24, and poverty peaks at 6.5% of the population at age 26.

- The uncertainty associated with a wage offer, and the incidence of marital dissolution appear to be principal drivers of over-indebtedness in the model.

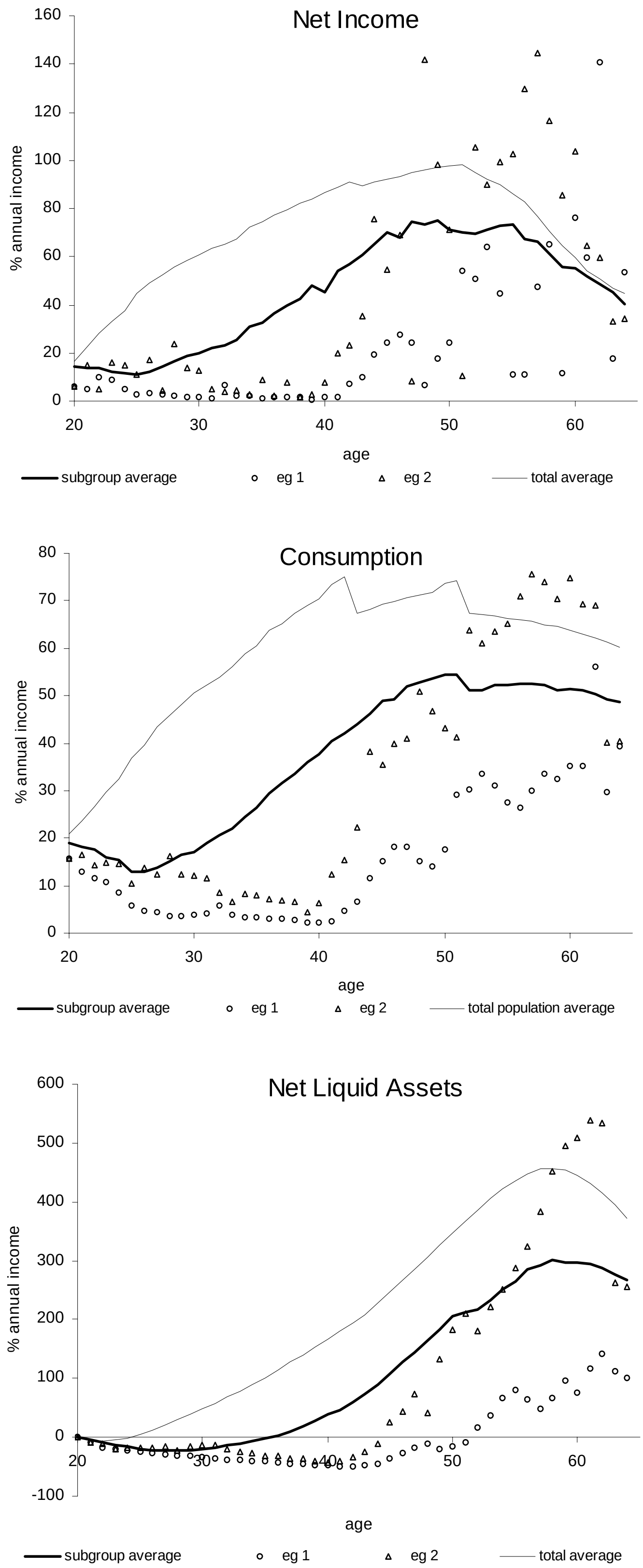
- Singles have a higher probability than otherwise similar couples of experiencing poverty or becoming over-indebted early in the simulated lifetime because they tend to have lower incomes on average, and more tenuous employment opportunities.

- The probability of over-indebtedness and poverty tend to be lower for households with dependent children than those without. This result arises because the tax and benefit system that is considered for analysis makes a larger adjustment in response to the presence of a child than the increase in consumption needs implied by the assumed equivalence scale. This result – in contrast with the empirical incidence of poverty in the population – should be seen in the context of the modelling assumptions, which include 100% benefit takeup and a family formation process that is independent of income.

- The rational agent model suggests that the Pension Guarantee is an effective policy tool for limiting the incidence of poverty prior to State Pensionable Age.

9 Summary and Conclusions

Private sector indebtedness has grown substantially over the last decade, and is the subject of increasing public concern. The accrual of private sector debt has, however, been undertaken in the context of an historically benign economic environment, which complicates any evaluation of whether, and in what ways, the recent financial decisions of households may have been imprudent. This report is designed to shed some light on this issue by exploring the interactions between household savings and indebtedness using an articulated and carefully calibrated rational agent model.



Notes: Author calculations based on simulated population with less than £50 per week consumption at some age between 20 and 64

Monetary values expressed as percentages of average annual gross household income between ages 20 and 64, equal to £45,757 in the simulations

Figure 26: Life profiles of households constrained at age 30

The rational agent model considered for analysis assumes that individuals make sensible financial decisions. Decisions regarding consumption and labour supply are considered to be made to take into account the prevailing economic environment, and expectations regarding the future. In this context, households take on unsecured debt when they expect their respective incomes to rise in the future, so that debt is a positive mechanism which facilitates income smoothing over the lifecycle. Importantly, the expectations assumed for decision making in the model are consistent with the processes that govern the intertemporal variation. Furthermore, the rational agent model has been specifically designed to capture key characteristics that have been identified by the empirical literature as having an important influence on household saving and indebtedness.

The study obtains a number of interesting findings. First, the model that is described here substantially under-states both non-pension wealth and unsecured debt reported for the private sector in the National Accounts. With regard to the understatement of non-pension wealth, we conjecture that the difference between the model and survey data could be resolved by including an explicit bequest motive in the model. In the case of unsecured debt, however, plausible magnitudes for the effects of augmenting the model to better reflect the complexity of the household's asset allocation problem suggest that the model would still understate the National Accounts aggregate by 30%. While this observation does not permit any strong conclusions to be drawn, it does suggest the need for further analysis.

Second, unsecured debt generated by the rational agent model tends to arise as the product of low incomes. Low incomes, as they bear upon debt, are found to arise as the primarily due to changes in marital status (and marital dissolution in particular), and the (exogenously imposed) probability of a low wage offer (included as a proxy for involuntary unemployment). These observations are qualitatively consistent with survey data, and emphasise the importance of "indirect drivers" of private sector indebtedness. Indeed, to the extent that individuals are unable to affect the probability that they suffer negative shocks of the type referred to here, one might consider there to exist a potential insurance role for government.

Third, the model suggests that the trend toward a higher educated population should result in greater demand for credit early in any given birth cohort's lifetime, and larger debts for those who are indebted. These effects are driven primarily by the higher wage growth with age that is commonly observed for higher educated, relative to lower educated individuals. If this result is found to accurately describe the practical reality in the future, then it may provide a motive to expand the role of student loan programmes.

Fourth, the rational agent model implies that approximately one fifth of all households should experience at least one year of over-indebted and poverty during their lives. Over-indebtedness and financial deprivation are observed from the model to peak in the mid 20s, and levels just under 10%

of the respective birth cohort, and again are found to be driven primarily by the receipt of a low wage offer or a marital dissolution. This result is important as it implies that *over-indebtedness and financial deprivation cannot simply be dismissed as a product of poor financial planning.* Singles without dependent children tend to have the highest probability of being over-indebted or experiencing a period of low consumption. This is, in part, driven by the fact that the tax and benefit system considered for analysis makes a larger adjustment in response to the presence of a dependent child than the coincident increase assumed for consumption needs. This finding should be taken in the context of the modelling assumptions which imply that all benefits are taken up and that lone parents are on average better off than is observed empirically.

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A Household level data for assets and debt from the British Household Panel Survey

The British Household Panel Survey (BHPS) provides detailed panel data for households reported at annual intervals between 1990/01 and 2004/05 (the most recent wave issued at the time of writing). The original sample, comprised of 10264 interviewed individuals drawn from 5538 households, was selected to be representative of the UK cross-sectional population in 1990/01.[56] The data provided by wave 10 of the BHPS, for 2000/01, are of particular interest here as they include responses to a suite of questions that were designed to describe household assets and debt.[57] When combined with the time-series aspect of the survey, these questions provide a valuable source of wealth data for the UK.

Wave 10 of the BHPS provides data for 8903 interviewed individuals from 4916 households. Including additions made to the original BHPS sample expands the number of household records to 8761. This sample was restricted for the current analysis to omit those households with self-employment income in excess of 10 percent of labour income, or those with more than two adults, to give a total sample of 6975 households. Responses of these households were used to obtain three measures of household wealth from the BHPS – housing wealth, securities wealth, and pension wealth. Each of these three measures of wealth are described in the following subsections. The data were calculated using programmes that are written for SPSS and GAUSS, which may be obtained from the authors upon request.

A.1 Housing wealth

Housing wealth refers to the value of a family's place of residence (for owner occupiers), and their second home (if they have one), net of any outstanding mortgage commitments.[58] The illiquid nature of wealth held in residential property complicates accurate evaluation for survey purposes. Various alternative methods can be devised to impute an 'expected' or 'reasonable' value of housing wealth from alternative data, such as the size of a household's residence, its region, and the value at purchase. The BHPS places the onus of such imputations upon the survey respondent by asking them: "About how much would you expect to get for your home if you sold it today?" Similarly, with regard to the value of mortgage debt the BHPS asked respondents: "Approximately how much is the total amount of your outstanding loans on all the property you (or your household) own, including your current home?"

It is clear that answers to these sorts of questions may be inaccurate, which (in the case of aggregate property value) the BHPS takes into consideration by asking respondents whether their expected value

[56] The first sample was drawn from the small users Postal Address file for Great Britain south of the Caledonian canal (excluding Northern Ireland). Due to the repeated survey methods employed, the most recent waves of the BHPS no longer provide a representative sample of the UK population. See Taylor (2005) for further details regarding the BHPS.

[57] The BHPS also includes wealth data for the 1985/86 wave, and is scheduled to provide similar data for 2005/06 in 2007.

[58] Details are drawn from responses made by the household reference person (definition provided below).

falls into one of a specific range of values. The BHPS also asks whether the property is owned jointly with a third party. Where shared ownership is identified (0.3 percent of respondents), 50 percent of the property value was assigned to the household for the analysis presented here.

A.2 Securities wealth

Wave 10 of the BHPS asked respondents to provide information regarding the value of any savings that they held in tradeable securities. Specific reference was made to the following: Savings or Deposit Accounts; National Savings Bank (Post Office); TESSA or ISA; National Savings Certificates; Premium Bonds; Unit Trusts / Investment Trusts; Personal Equity Plan; Shares (UK or foreign); National Savings Bonds; or Government or company securities. Individuals were asked to reveal the value of their aggregate savings in these products, and whether they were owned jointly or in their sole name. Where joint holdings were identified, the analysis presented here assumes that 50 percent of the associated value was owned by the responding individual.

Importantly, no explicit reference was made by interviewers to account statements or listed market values. Rather, respondents are asked questions such as: "Thinking now about the investments you have ... about how much is the total value of these investments?" The form of this question is consistent with that asked for housing, and focusses upon perceived (rather than actual) household wealth.

The BHPS also asks respondents to provide details about any debts that they owe. With regard to financial wealth, the relevant questions make specific reference to the following: Hire Purchase Agreements; Personal Loans; Credit Cards; Catalogue or Mail Order Purchase Agreements; DSS Social Fund loan; Overdrafts; Student Loan; any other loans for a private individual (not including mortgages).

The total value of assets and loans for a household were calculated by summing the individual holdings recorded by the BHPS for any adult members of the household.

A.3 Pension wealth

Household wealth held in the form of pension entitlements is highly opaque in the UK, which has resulted in a low level of public awareness regarding the provisions made for retirement and how such provisions affect future standards of living. Nevertheless, pension savings have a prominent impact on the circumstances of individuals over the life course, and should consequently be taken into consideration in a study such as this.

In spite of their opacity, measures of pension wealth can be imputed using a series of variables that are described by the BHPS. The methodology used here is similar to Blundell et al. (2002), and focuses upon rules that were commonly applied by the various schemes from which the UK pension system was comprised in 2000/01. Three pension schemes are considered for the imputation of pension wealth;

Occupational Pensions (OP), Personal Pensions (PP), and the State Earnings Related Pension Scheme (SERPS). The basic state pension and the invalidity benefit, which are also addressed by Blundell *et al.*, are omitted here because they are treated separately in the tax and benefit routine of the simulation model that is used for analysis.

The imputation method that was used to impute pension wealth involves two distinct stages. In the first stage, the annuity associated with alternative pension entitlements was determined, and in the second, the lifetime present value of the annuities identified in the first stage was calculated. Each of these stages is discussed below.

Annuity value of pension entitlements

The BHPS data were first checked to determine whether an individual is identified as receiving a pension in 2000/01. If so, then imputations are unnecessary, since the pension annuity is described explicitly by the data (as a weekly benefit received). If, however, an individual is not identified as drawing upon a retirement pension, then the value of their current entitlement to a future pension must be imputed.

Between 1978 and 2001, an individual could be a member of three alternative pension schemes that relate retirement benefits to income earned during the working lifetime. Primarily, an individual could choose to remain in SERPS, a State administered scheme, or to opt-out of the state system and instead contribute to an OP and/or a PP.

The imputation procedure used to obtain the measures of pension wealth discussed here begins by checking whether a given individual was employed in 2000/01, and if so, whether they were a member of an OP and/or a PP. Individuals who are identified as contributing to an OP and/or a PP in 2000/01, are assumed to have been members of their respective pension scheme(s) for the entire period during which they were employed in the position held during 2000/01 (and hence to have opted out of SERPS). The annuity associated with a 2000/01 entitlement to a future OP, A_{OP}, is obtained by:

$$A_{OP} = \chi \left(PE_{2001} - \beta LEL_{2000} \right) N \tag{9}$$

where χ is the accrual rate (taking a value of 1/60 for employees in the private sector, and 1/80 for employees in the public sector), PE_{2001} is the pensionable income (best salary for the period 1997-2000 for employees in the private sector, and for the period 1990-2000 for the public sector), β is the integration factor (equal to one for the private sector and zero for the public sector), LEL_{2000} is the Lower Earning Limit associated with National Insurance Contributions (NICs) for 2000/01, and N is the number of years of membership. Equation (9) is based upon the most common OP schemes that were offered up to 2000/01, which took the form of Defined Benefit Plans.[59] Since the BHPS provides

[59] See Blundell *et al.* (2002, p. C163).

details regarding the income history of individuals for the ten year period between 1990/01 and 2000/01, the above assumptions imply that A_{OP} can be calculated for each individual without further imputation or assumptions. In the absence of alternative criteria, equation (9) is also used to calculate the annuity value of current entitlements to a future PP, A_{PP}.

Membership of SERPS is assumed for any individual who is identified as employed in 2000/01 and not a member of an OP or a PP, and for any alternative employment held prior to 2000/01. Hence, an individual who is identified as holding one position between 1985 and 1993, and another position from 1994 to 2000, where they were also identified as a member of an OP in 2000/01, would be treated as if they were a member of an OP for 7 years (between 1994 and 2000), and a member of SERPS for 9 years (between 1985 and 1993). No membership is assumed for any pension scheme prior to the period for which some information is provided by the BHPS. In the example described above, membership can be identified for the period prior to 1990 because the first wave of the BHPS (undertaken in 1990) provides data on pre-standing employment tenure. It is important to bear in mind that this methodology is likely to under-predict pension wealth. Focusing only upon the most recent employment tenure for calculating OP and PP omits any OP or PP entitlements accrued under a previous employer. This effect is (at least partially) offset by the assumption that pension rights are accrued under SERPS for the remaining employment history described by the BHPS. Any pension entitlements accrued during employment in positions held prior to the period described by the BHPS are omitted from the analysis (unless the household is identified as receiving an associated pension benefit in 2000/01).

The annuity value associated with a future SERPS pension entitlement, A_S, is obtained by:[60]

$$A_S = \sum_{t=\max(r,1978)}^{2000} \frac{\theta_{R,t}\left\{\phi_{2000,t}(x_t - LEL_t)\right\}}{[R-1-\max(r,1978)]} \tag{10a}$$

$$x_t = \max\left[\min\left(y_t, UEL_t\right), LEL_t\right] \tag{10b}$$

where θ_t is the accrual rate (which depends upon the year, t, and the year of reaching state pensionable age, R), $\phi_{2000,t}$ is an index that adjusts earnings to the levels observed for 2000/01, r is the year an individual reaches age 16, y_t is an individual's income in year t, and UEL_t is the Upper Earning Limit associated with NICs for year t. The values of $\phi_{2000,t}$, LEL and UEL used are defined by legislation; for full details see the *Retirement Pensions Guide*, NP46 from January 2001, DSS Communications: Leeds.

The specification of equation (10a) indicates that the annuity associated with any future SERPS entitlement depends upon the complete income history of an individual while they were contributing to the scheme. The BHPS records the income earned by an individual for each year between 1990 and 2000. It also defines how long an individual had been in the employment that they held in 1990, but

[60] Note that typographical errors are associated with the equation provided by Blundell *et al.* (2002, p. C161).

does not provide details regarding income earned prior to 1990. To identify the annuity value of SERPS entitlements that were accrued prior to 1990, it is consequently necessary to impute measures of income.

Two equations of earnings by age were estimated to impute individual income prior to 1990, one for higher educated (full-time education beyond the age of 18), and another for lower educated individuals. The estimates were obtained using data for household reference people between ages 18 and 70 years, as reported by the 2000/01 Family Expenditure Survey (1041 observations for the higher educated, and 4476 for the lower educated subgroup). The earnings equations were estimated using a sample selection model of individual full-time employment wages. This model takes into consideration the fact that wages are only observed for individuals who are working, and that there is likely to be a relationship between the probability of an individual working and their wage rate. The regressions were undertaken using the "Sampsel" procedure in TSP; for full details refer to the "TSP 4.4 User's Guide" (see http://elsa.berkeley.edu/wp/tsp_user/tspugpdf.htm).

The sample selection model involves estimating two equations, a probit to identify individuals who are employed, and a (log) wage equation:

$$\begin{aligned} emp_i &= \pi_0 + \pi_1 age_i + \pi_2 age_i^2 + \pi_3 mar_i + \pi_4 chu5_i + \\ &\quad \pi_5 ch5p_i + \pi_6 na_i + \pi_7 totx_i + \varepsilon_{1i} \qquad (11) \\ \ln(y_i) &= \beta_0 + \beta_1 age_i + \beta_2 age_i^2 + \beta_3 age_i^3 + \varepsilon_{2i} \qquad (12) \end{aligned}$$

where:	emp_i (a220+a221>30)	identifies when individual i works in excess of 30 hours per week
	age_i (a005)	age of individual i
	mar_i (a006= 1 or 2 or 3)	identifies when individual i is married
	$chu5_i$ (a040+a041)	number of children under 5 years old
	$ch5p_i$ (a042)	number of children 5 years or older
	na_i (a049-chu5-ch5p)	the number of adults in i's household
	$totx_i$ (p550tp)	total expenditure of i's household
	y_i (p008+p011+p037+p047)	normal weekly wage of individual i

Bracketed terms identify FES variable codes

Table 8 reports regression statistics derived for the sample selection model described above. These statistics indicate that all of the estimated coefficients are highly significant. Two variables were omitted from the regressions due to low statistical significance, one from each of the lower and higher educated subgroups. The implied profiles of income with age are displayed in Figure 27.

In addition to the wage profiles obtained from the sample selection model, Figure 27 also displays

Table 8: Sample Selection Model of Weekly Wages

	higher educated		lower educated	
		Probit Regression		
C	-2.57042	(0.66788)	1.22975	0.097911
AGE	0.18411	(0.03137)	-0.039011	1.79E-03
AGE^2	-0.00264	(0.00035)		
MAR	0.50230	(0.12280)	6.85E-01	4.60E-02
CHU5	-0.18574	(0.09221)	-0.239085	0.043078
CH5P	-0.12404	(0.04727)	-0.136978	0.022174
NA	-0.15515	(0.06866)	-0.190953	0.027165
TOTX	1.13E-03	(7.06E-5)	2.01E-03	3.41E-05
		Wage Equation		
C	4.79705	(0.35930)	3.69E+00	6.75E-01
AGE	0.06895	(0.01726)	0.13435	0.047165
AGE^2	-6.34E-04	(2.01E-4)	-2.37E-03	1.07E-03
AGE^3	-	-	1.46E-05	7.82E-06
SIGMA	0.67712	(0.01393)	7.32E-01	5.03E-03
RHO	-0.83633	(0.02360)	-0.890276	5.64E-03

standard errors in parentheses

SIGMA = standard deviation of the wage regression

RHO = correlation between the errors of the probit and wage equations

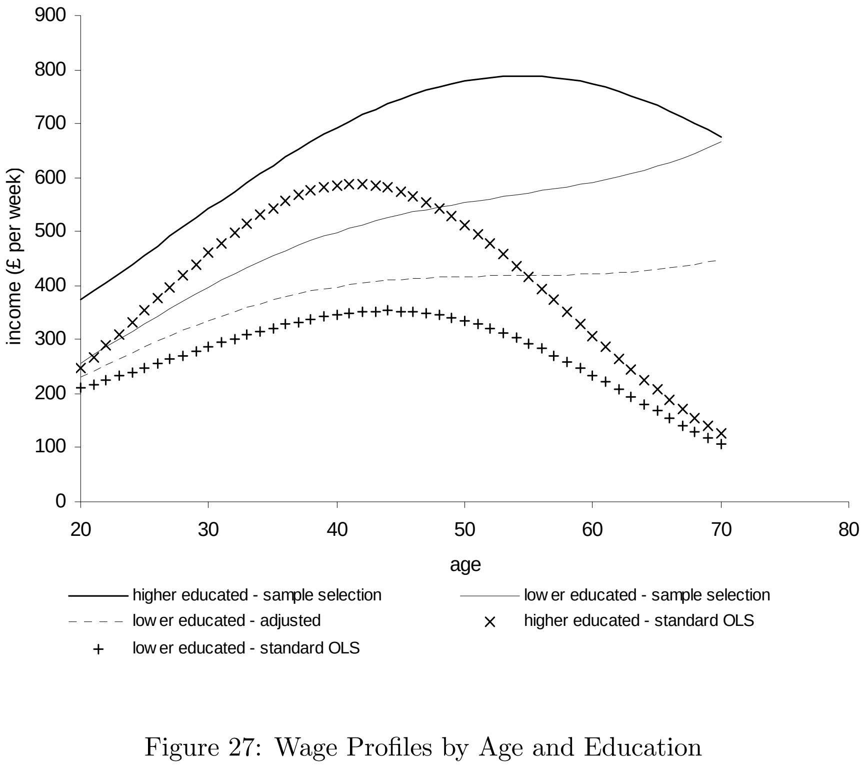


Figure 27: Wage Profiles by Age and Education

profiles estimated by standard Ordinary Least Squares (OLS). The profiles displayed in the figure indicate that the adjustment for sample selection has a small effect on the estimates obtained at low ages relative to high ages, which is consistent with the early retirement of individuals who had previously earned relatively high incomes. The effect is, however, surprisingly large for the lower educated population, a result that is attributable to the diminishing power of education as an identifier for the high wage subgroup as age increases. Reducing the estimate obtained for β_1 by 12 per cent of its standard error alters the profile of wages for the lower educated subpopulation to the series denoted "lower educated – adjusted" in Figure 27. The estimates derived from the sample selection model are used to project incomes for the higher educated, while the adjusted profile is used for the lower educated.

In each year prior to 1990, an individual's projected income, $\tilde{y}$, is given by:

$$\tilde{y}_{i,t} = \frac{\rho_i \hat{y}\left(age_{i,t}\right)}{\phi_{2000,t}} \tag{13a}$$

$$\rho_i = \sum_{s=1990}^{2000} \frac{y_{i,t}\phi_{2000,s}}{\hat{y}\left(age_{i,s}\right) m_i} \tag{13b}$$

where $y_{i,t}$ is the income recorded by the BHPS for individual i in year t, $\hat{y}\left(age_{i,t}\right)$ is the value of income derived from equation (11), given the age of individual i in year t and the parameter values discussed above, m_i is equal to the number of years that $y_{i,t} > 0$ between 1990 and 2000, and all other variables take their previously stated definitions.

Given an individual's age, education status and pension scheme contributions in 2000/01, and given their labour force status and income history described by the BHPS, the procedures described above enable the current annuity value of pension rights to be calculated.

Lifetime present value of pension annuities

The lifetime present value of the respective pension annuities described above, PW_i, are calculated by:

$$PW_i = \sum_{t=2000}^{t_u} \pi^{t-2000} \delta_t A_i \tag{14}$$

where t_u is the last year in which the household may receive pension annuity i, π is the real annual growth rate of pension benefits (assumed to be 2 percent), δ_t is the probability that a household will receive pension annuity i in year t, and A_i is the annuity value described in the previous subsection. The probability that a household will receive a particular pension annuity in year t depends upon:

- the age of the person who accrued the pension rights in year t – it is assumed that all pension rights will be received from state pensionable age (60 for women and 65 for men), if the BHPS data do not indicate earlier receipt.

- the pension rights identified for an individual's spouse – in the case of each of the three pension schemes defined above the spouse is assumed to share pension entitlements if they are ineligible for a pension of their own.

- the probability that an individual with a pension entitlement will survive up to a given year – assumed to depend upon their age and sex (based upon statistics reported for the period 2000-02 by the UK Government Actuaries Department).

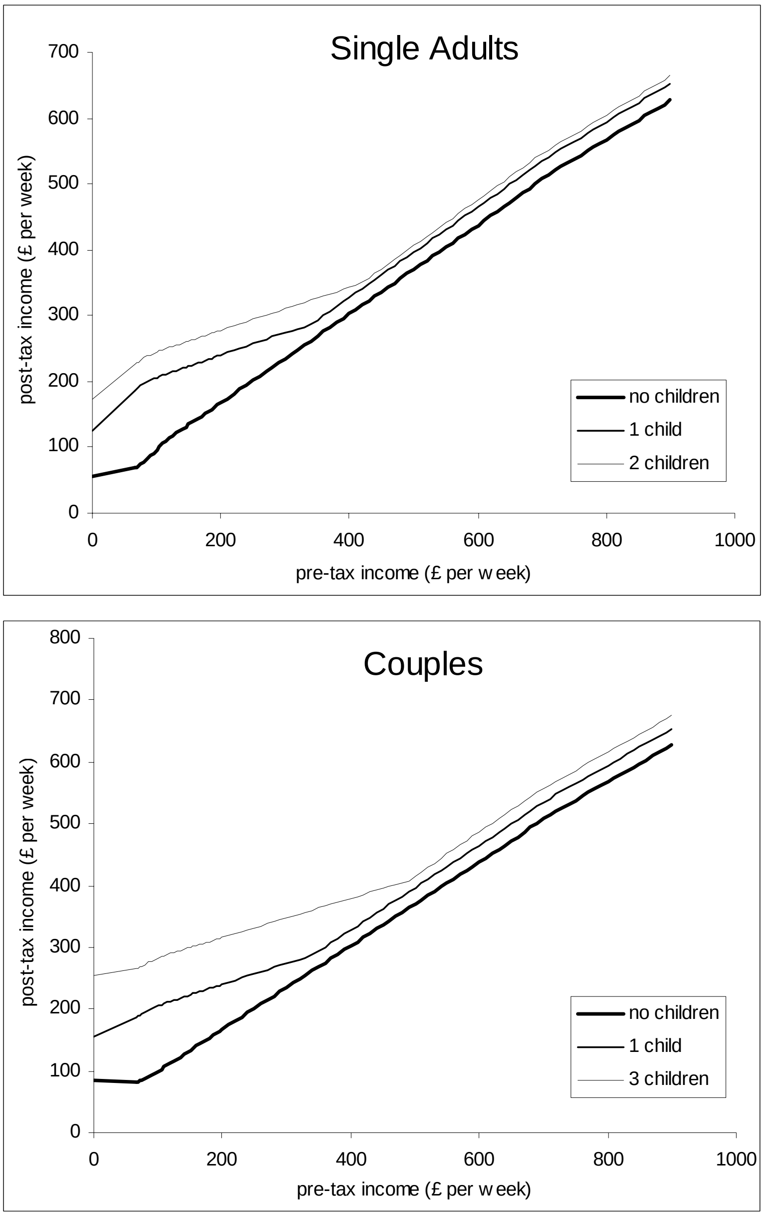
B Simulated Tax and Benefits Policy

The simulations are based upon a parameterisation of the tax functions described in Section 3.2, which were arrived at by the following procedure. First, for each practicable combination of numbers of adults and children in a household, disposable (post-tax and benefit) income was identified as a function of private (pre-tax and benefit) income. This function was then summarised by a four piece linear spline. Intersections between the linear segments of the spline were smoothed to omit discontinuities in the marginal tax rate.[61] This produced a set of parameters that describe the tax function of each household type, distinguished by the number and age of household members. The parameters so obtained were themselves summarised as a function of the numbers of adults and children in a household, and these summaries are used by the model to generate the effects of transfer policy. Examples of the functions applied for analysis are reported below.

C Calibration Statistics for Education Specific Subgroups

D Additional Contour Maps of Over-Indebtedness and Poverty

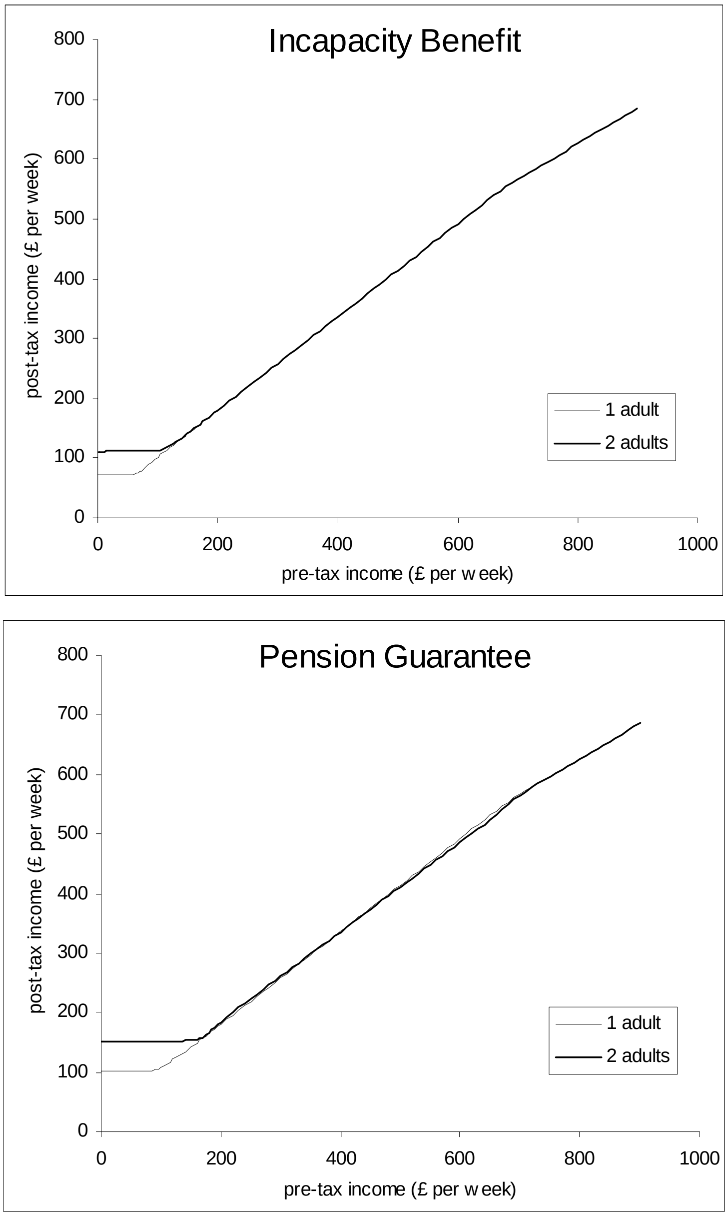
[61] The linear segments were smoothed using a Boltzmann function, which produces a sigmoidal curve.



Source: Stylised characterisation of profiles described by Tax Benefit Model Tables

The authors may be contacted for further details

Figure 28: Simulated Tax Functions – Working Lifetime



Source: Stylised characterisation of the IB and Pension Guarantee

The authors may be contacted for further details

Figure 29: Tax Functions for Early and Later Retirement

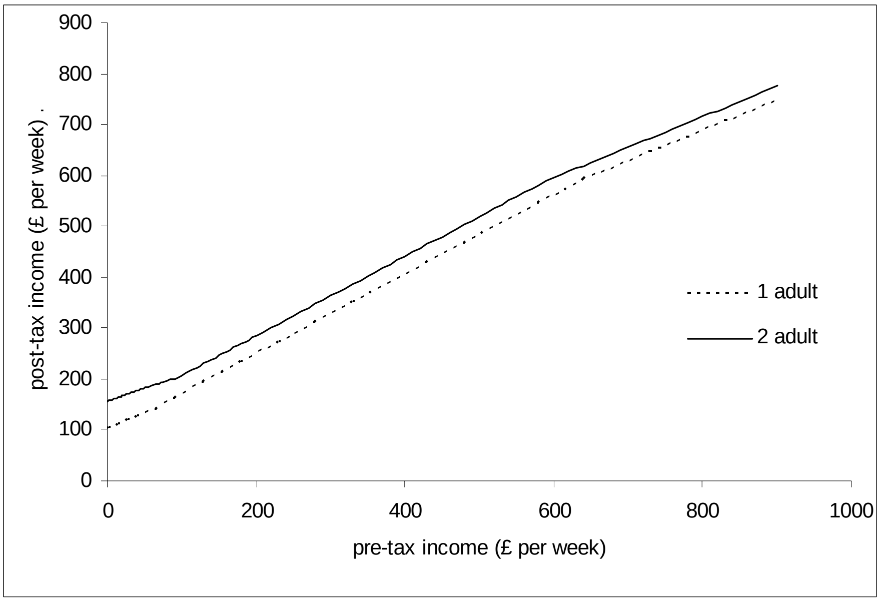
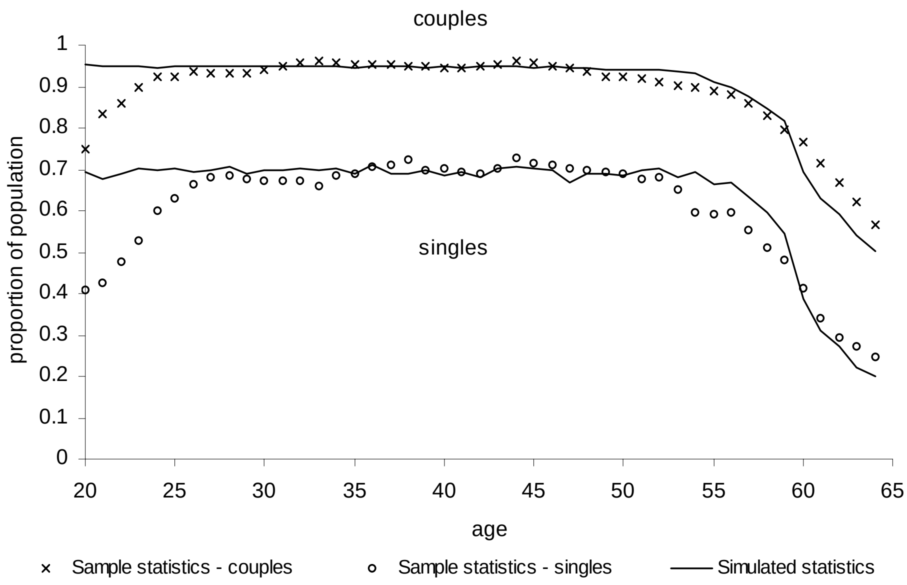
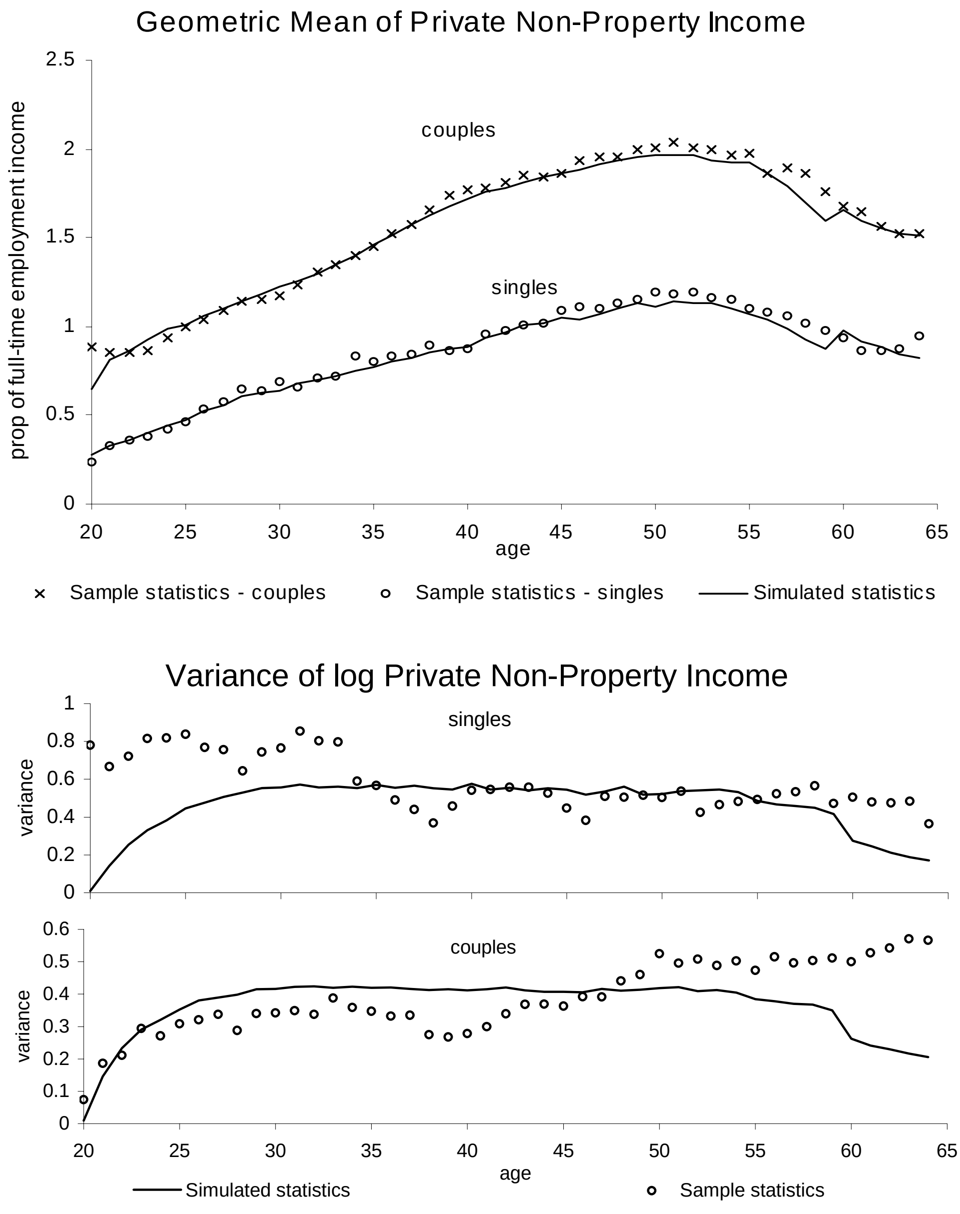


Figure 30: Tax Functions for Retirement



Notes: Sample statistics – age profiles calculated from Expenditure and Food Survey data, 2001/02, 2002/03, and 2003/04
Simulated statistics – age profiles generated from model, using calibrated parameters reported in Table 4
ELSA – percentage of men aged 55-59 economically inactive by wealth quintile, 2002/03 wave of the English Longitudinal Study of Aging, Table 4A.12, Marmot et al. (2003)

Figure 31: Timing of Retirement – simulated versus survey data for lower educated households

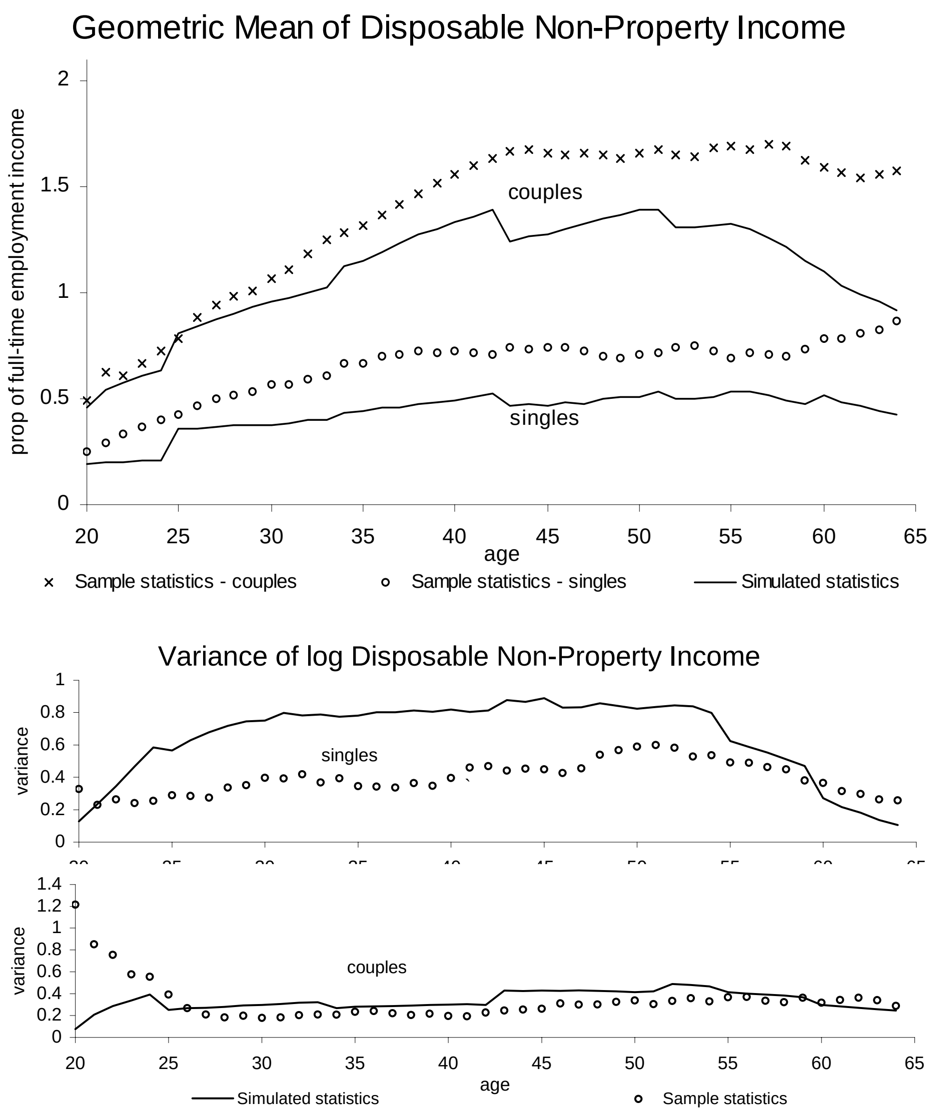


Notes: Sample statistics – age profiles calculated from Expenditure and Food Survey data, 2003/04

Simulated statistics – age profiles generated from model, using calibrated parameters reported in Table 4

Monetary values reported as proportions of average annual full-time employment income for 2003/04, equal to £22,724

Figure 32: Private Non-Property Income Profiles by Age – simulated versus survey data for lower educated households

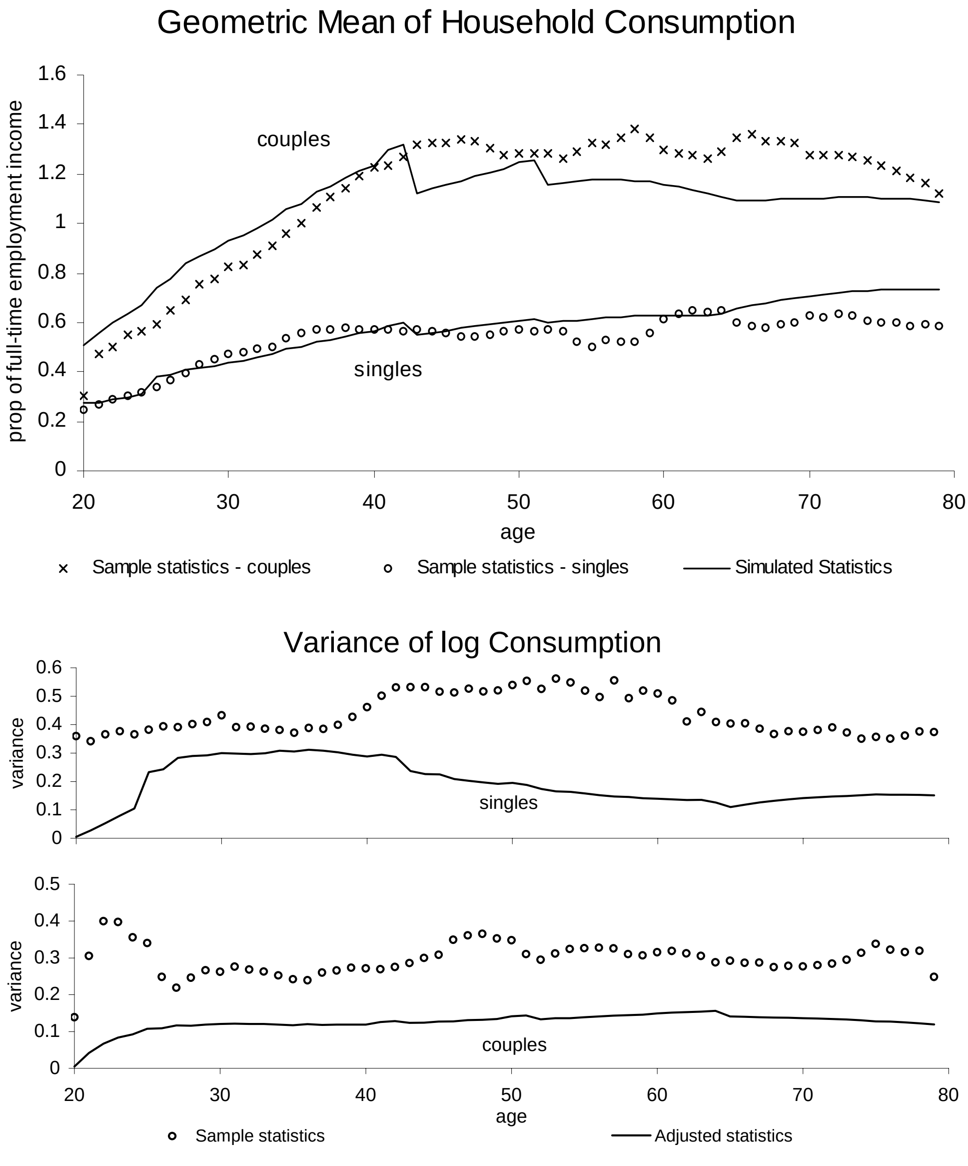


Notes: Sample statistics – age profiles calculated from Expenditure and Food Survey data, 2003/04

Simulated statistics – age profiles generated from model, using calibrated parameters reported in Table 4

Monetary values reported as proportions of average annual full-time employment income for 2003/04, equal to £22,724

Figure 33: Disposable Non-Property Income Profiles by Age – simulated versus survey data for lower educated households

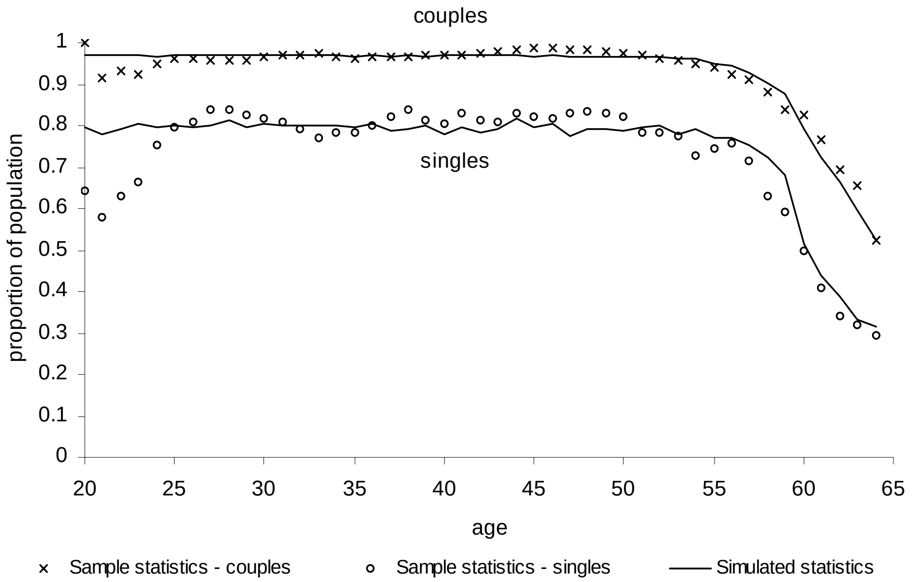


Notes: Sample statistics – age profiles calculated from Expenditure and Food Survey data, 2003/04

Simulated statistics – age profiles generated from model, using calibrated parameters reported in Table 4

Monetary values reported as proportions of average annual full-time employment income in 2003/04, equal to £22,724

Figure 34: Consumption Profiles by Age – simulated versus survey data for lower educated households

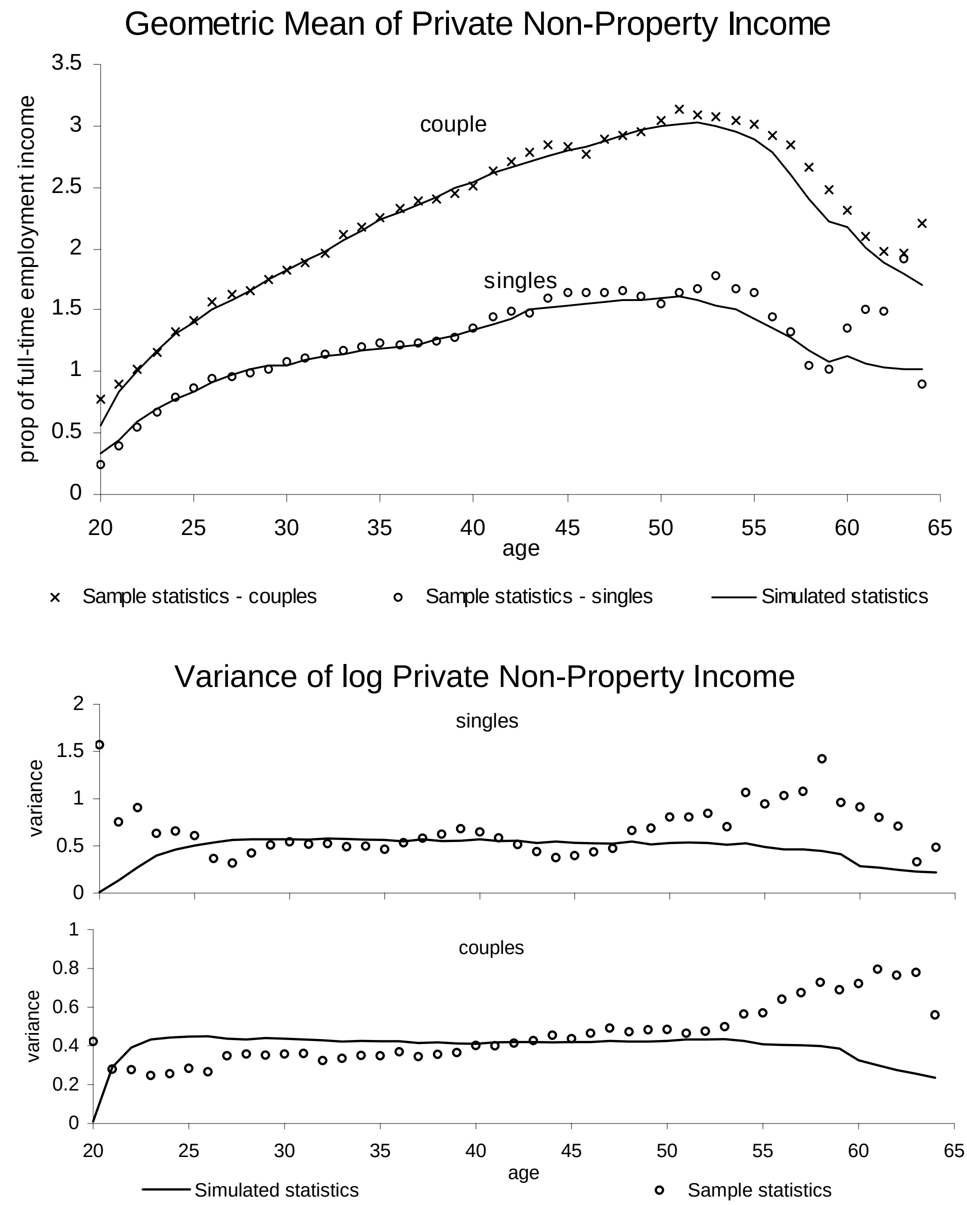


Notes: Sample statistics – age profiles calculated from Expenditure and Food Survey data, 2001/02, 2002/03, and 2003/04

Simulated statistics – age profiles generated from model, using calibrated parameters reported in Table 4

ELSA – percentage of men aged 55-59 economically inactive by wealth quintile, 2002/03 wave of the English Longitudinal Study of Aging, Table 4A.12, Marmot et al. (2003)

Figure 35: Timing of Retirement – simulated versus survey data for higher educated households

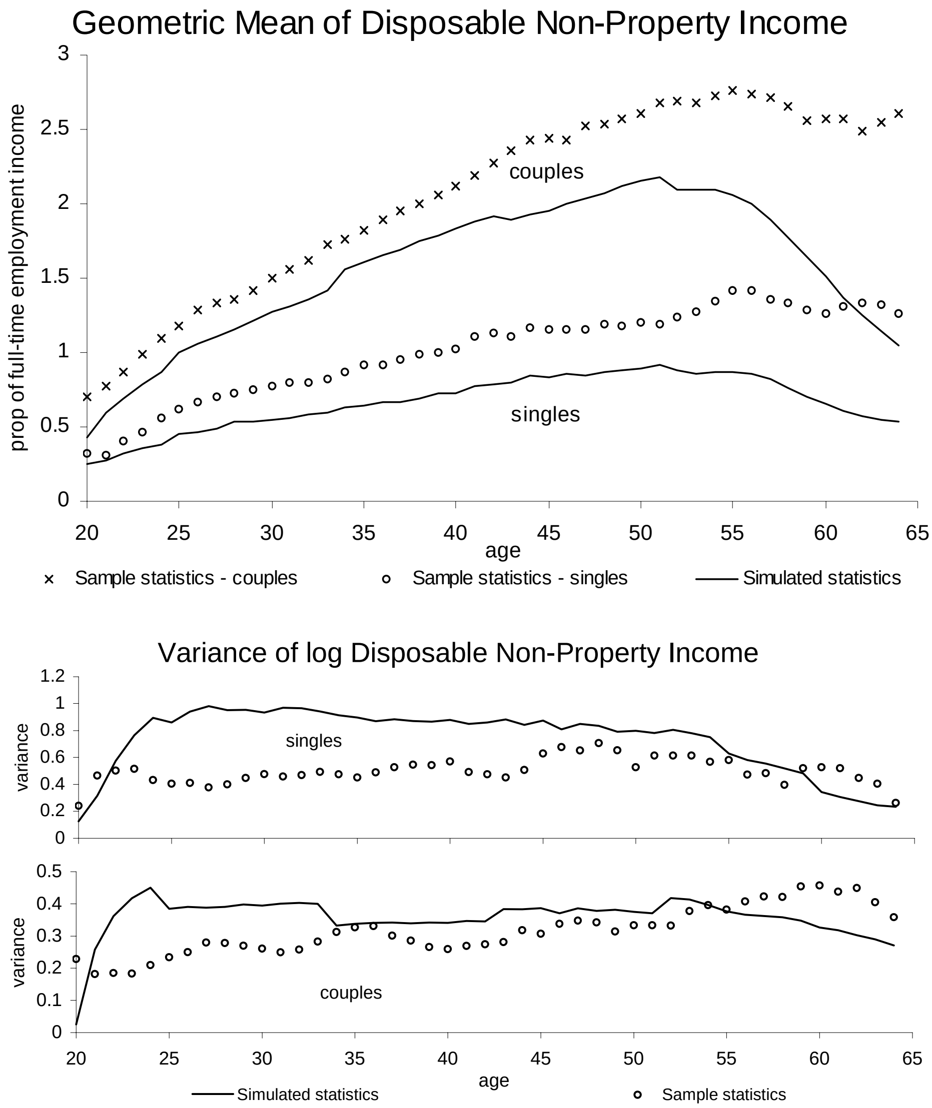


Notes: Sample statistics – age profiles calculated from Expenditure and Food Survey data, 2003/04

Simulated statistics – age profiles generated from model, using calibrated parameters reported in Table 4

Monetary values reported as proportions of average annual full-time employment income for 2003/04, equal to £22,724

Figure 36: Private Non-Property Income Profiles by Age – simulated versus survey data for higher educated households

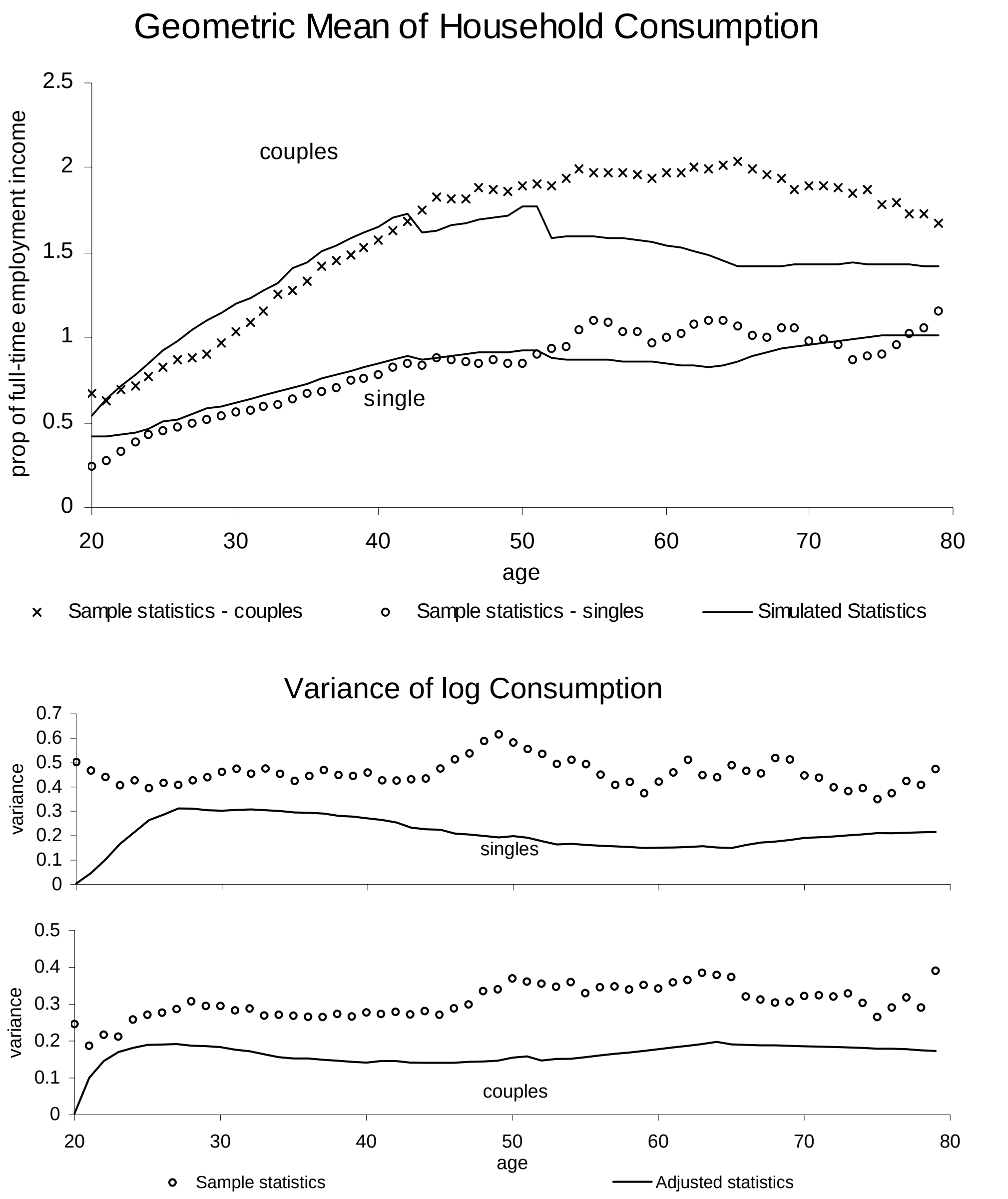


Notes: Sample statistics – age profiles calculated from Expenditure and Food Survey data, 2003/04

Simulated statistics – age profiles generated from model, using calibrated parameters reported in Table 4

Monetary values reported as proportions of average annual full-time employment income for 2003/04, equal to £22,724

Figure 37: Disposable Non-Property Income Profiles by Age – simulated versus survey data for higher educated households

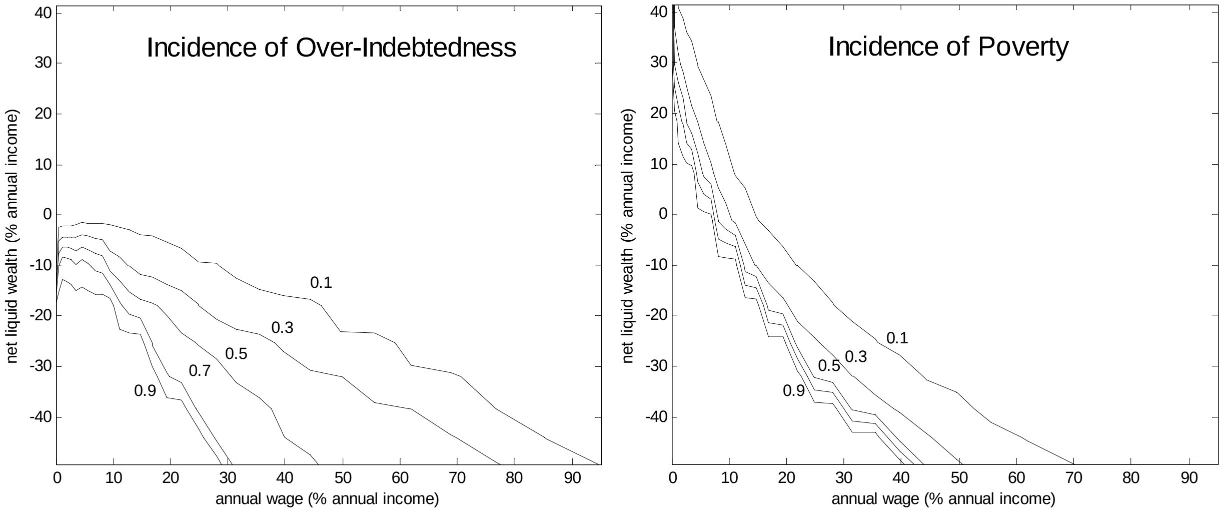


Notes: Sample statistics – age profiles calculated from Expenditure and Food Survey data, 2003/04

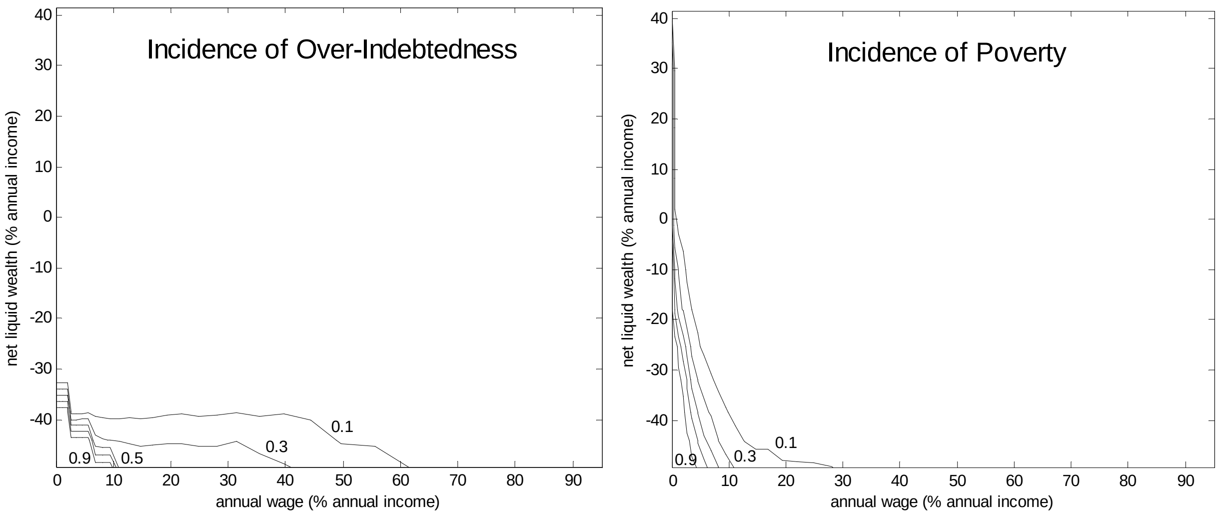
Simulated statistics – age profiles generated from model, using calibrated parameters reported in Table 4

Monetary values reported as proportions of average annual full-time employment income in 2003/04, equal to £22,724

Figure 38: Consumption Profiles by Age – simulated versus survey data for higher educated households



Panel A: single adults with no dependent children



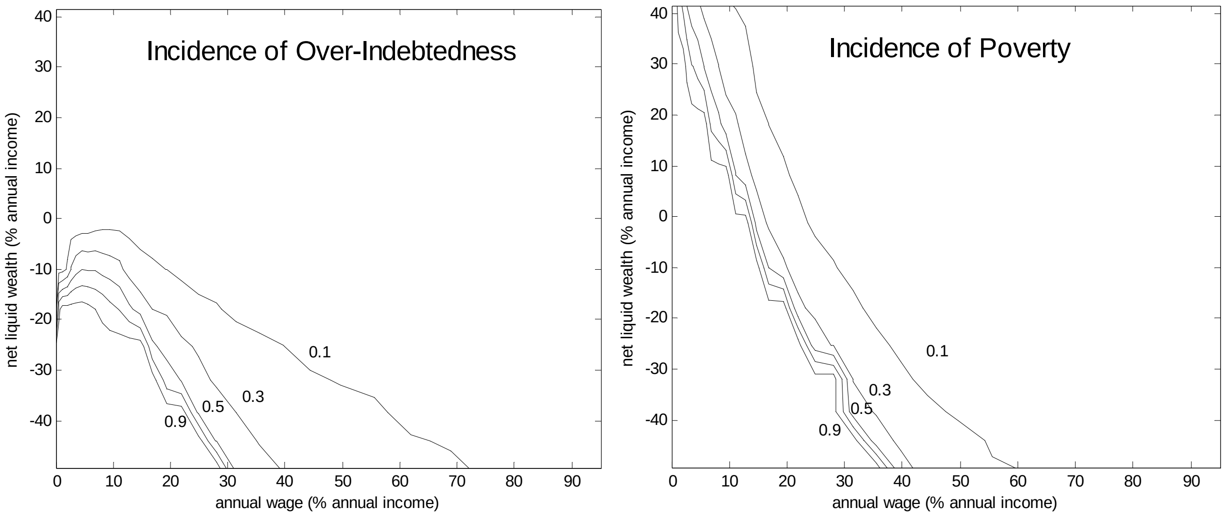
Panel B: single adults with two dependent children

Notes: Contour maps generated from model, using calibrated parameters reported in Table 4

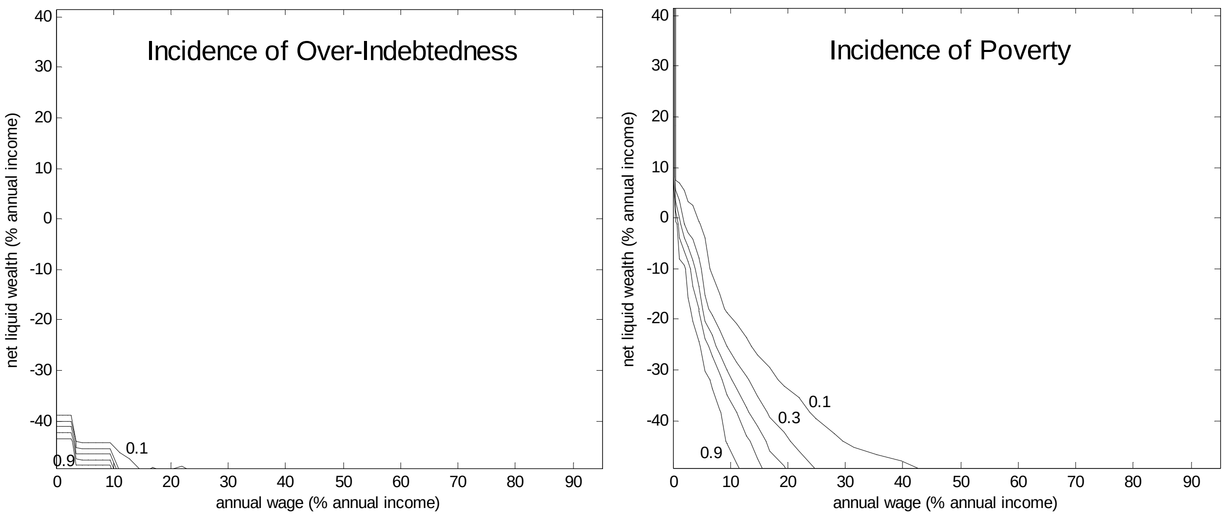
For households with 2 dependent children: first child born at age 25, second child born at age 34

Monetary values expressed as percentages of average annual gross household income between ages 20 and 64, equal to £45,757 in the simulations

Figure 39: Probability of Being Over-Indebted or Poor in at least One Year Between Ages 34 and 39 – contour maps by wage and wealth at age 34



Panel A: couples with no dependent children



Panel B: couples with two dependent children

Notes: Contour maps generated from model, using calibrated parameters reported in Table 4

For households with 2 dependent children: first child born at age 25, second child born at age 34

Monetary values expressed as percentages of average annual gross household income between ages 20 and 64, equal to £45,757 in the simulations

Figure 40: Probability of Being Over-Indebted or Poor in at least One Year Between Ages 34 and 39 – contour maps by wage and wealth at age 34